M. S. Molodensky

In Memoriam

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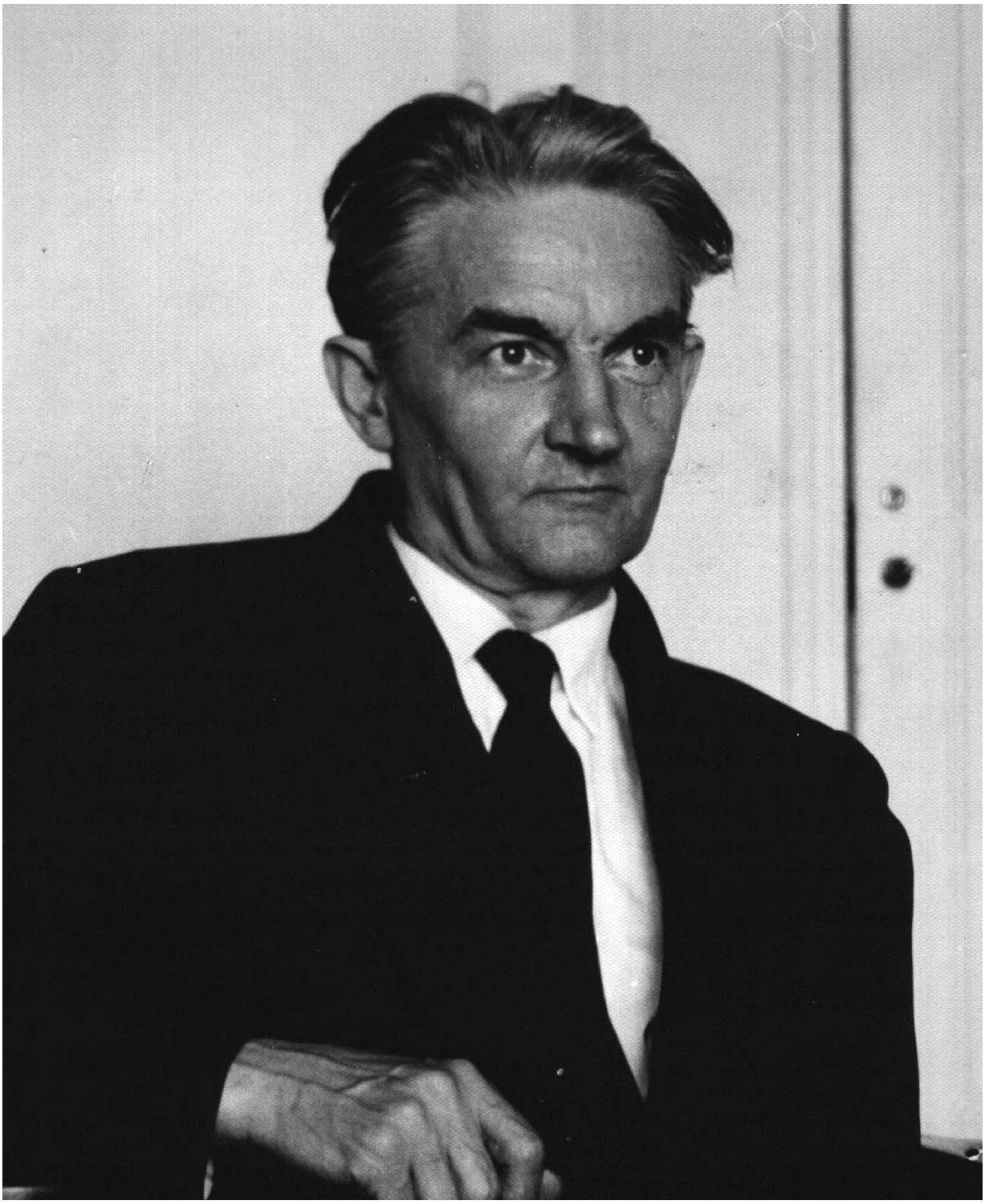

Foreword

This publication has met with considerable interest and is now out of print. Therefore I have decided to put it on my science page www.helmut-moritz.at in PDF for free download, as in all other works on this page, with kind permission of Prof. Bernhard Hofmann-Wellenhof, Director of the Institute of Navigation and Satellite Geodesy.

Particular thanks are due to Ruth Hödl and Norbert Kühtreiber for help in the non-trivial task of bringing the publication into electronic form.

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Preface

The year 2000 seems to be particularly appropriate to honor the greatest theoretical geodesist of the 20th century, Mikhail Sergeevich Molodensky (1909 – 1991). He has been a legendary personality, not only because of his scientific achievements — he is probably the only geodesist who would have deserved a Nobel Prize — but also because he was leading the reclusive life of an ascetic to whom only a very small circle of devoted friends had access. Mrs. M. I. Yurkina was one of these privileged persons, perhaps the most faithful of his disciples, if I may use this term usually reserved to religious–philosophical thinkers such as the mythical Pythagoras.

The present volume includes obituaries and recollections of friends and fellow Russian scientists. They provide a unique insight into life and work of a real genius, who was as great in his brilliant scientific ideas as in his simple modest life. They are also permitting a unique glance at life in the Soviet Union.

Through his work, M. S. Molodensky attracted many people to the austere beauty of physical geodesy, including myself. Although I had never met Molodensky, his ideas pervaded my whole scientific life. Therefore it is one of the greatest honors in my life that now I am able, by being coeditor of this volume together with Mrs. Yurkina, to help make his scientific and human personality better known to geodesists and geophysicists all over the world.

For this purpose it is very important to have all the material in English. Out of respect of the great language in which M. S. Molodensky thought, spoke and wrote, one of Molodensky's last articles is given in the Russian original, together with the English translation.

My original intention was to translate the material myself, but I could not find the time and energy. So very kindly, my Russian friends, on the initiative of Mrs. Yurkina, provided me with good English translations, sometimes even expanded with respect to the original. The translators are mentioned with gratitude: N. M. Molodenskaya, I. V. Stadnik, and P. V. Shvetsov, on the initiative of G. V. Demianov. All I had to do was to convert the texts into the pidgin English of international science, which was quite easy.

So to speak, Mrs. Yurkina and their Russian colleagues provided the cake, I only did the sugar–coating. She and the other authors deserve recognition for all that is good; for all the mistakes, the responsibility is mine.

Finally, I wish to thank warmly and sincerely all my Russian colleagues for their constant friendship and help. I am grateful to the authors, translators, and editors, especially of the journal "Geodeziya i Kartografiya", for the permission to reprint and translate several papers. The translation project was sponsored by the Russian Fund for Fundamental Researches No. 00–05–64284. I am also grateful to my friend Milan Burša in Prague for encouragement and help. Last but not least, my secretary Ruth Hödl deserves thanks for her painstaking editorial work.

December 2000 Helmut Moritz

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In Memoriam

Mikhail Sergeevich Molodensky

(Geodeziya i kartografiya, No. 2, pp. 58–59, 1992, Moscow, translated from Russian)

The prominent geodesist and geophysicist, a winner of two State Prizes and the Lenin Prize, M. S. Molodensky died on November 12, 1991 at the age of 82. He has entered into the history of science as a reformer of the theory of figure of the Earth; the study of Earth rotation and oscillations is bound to be associated with the name of M. S. Molodensky as well. The results of the researches made by M. S. Molodensky have created the basis for the astronomic–gravimetric leveling of the country, carried out for the astronomic–geodetic net: He also contributed to the development of the country's construction of gravimetrical instruments.

M. S. Molodensky was born on June 15, 1909 in the small town of Epiphan, province of Tula, in the family of a priest. He graduated from the Astronomic Department of the Mechanics and Mathematics Faculty of the Moscow State University in 1931 and was qualified by the diploma as a research worker and a lecturer. He worked with Academician V. G. Fesenkov for some years, then he was invited by F. N. Krasovsky to join the staff of the Central Research Institute of Geodesy, Aerophotogrammetry and Cartography (TsNIIGAiK) where he began working at the theory of pendulum co–swinging and the theory of the unregularized figure of the Earth.

M. S. Molodensky was a man of exceptional mathematical gifts. While solving problems raised by himself he could see the very essence of them, managing to preserve the minimum of conditions necessary for their solution. When speaking with him one could not help admiring and being fascinated by the brilliance of his ideas concerning even very practical tasks. He was able to foresee the direction of the development of geodesy for many years ahead.

In everyday life he was good–hearted, gentle, and always willing to render assistance to people. Always fully absorbed with his very original and profound thoughts, he was really a man "not of this world" [*]), which created some distance between him and other people. But all those who happened to know him thank destiny for this fortune.

For more than 25 years (1933–1960), M. S. Molodensky worked at TsNIIGAiK, combining this job with one at the Institute of the Physics of the Earth, where under the persistent pressure he agreed to become the director in the sixties. However, the administrative work undermined his health. Having abandoned the position, he continued to work for the Institute as a scientist.

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*) "not of this world". The basic bible. Containing the old and new testaments in basic English. E. P. Datton and Co. INC. N.Y. 1950. St. John, p. 778.

The theory of M. S. Molodensky has been recognized all over the world, his theory is included into students' textbooks and manuals. His scientific researches have not been fully appreciated so far, and not everything has been understood to the full extent. It mainly concerns studies on oscillations and rotation of the deforming Earth. The researches made by M. S. Molodensky have entered forever into the world's golden treasure of science. New generations of scientists will return to this source.


Июль 1938 г.

Mikhail Sergeevich Molodensky –– Life and Work

V. V. Brovar, M. I. Yurkina

(Translated from : Nauchno–tekhnicheskii sbornik po geodezii, aerokosmicheskim syemkam i kartografii, “Fizicheskaya geodeziya”, book 1, 11–39, TsNIIGAiK, Moscow. In Russian.)

Mikhail Molodensky — the prominent geodesist and geophysicist — spent a long and creative life, leaving after himself a great scientific heritage, as well as disciples and followers.

Mikhail Sergeevich was born on June 15, 1909 in the small town of Epiphan, Tula province, in the family of Sergey Mikhailovich and Nadezhda Mikhailovna Molodensky. He died on November 12, 1991 at the age of 82 years. His father was a priest. There were six children in their family: Nikolay, Maria, Sergey, Vladimir, Mikhail and Elena.

According to the journal of the Tula eparchy, Sergey Mikhailovich Molodensky graduated from Tula Spiritual Seminary with first degree in 1897. He then served for one year as a psalmist of the Dimitri cemetery church in Tula. In 1899 he was raised to the rank of priest of the village Smorodino of the Epiphan district. In 1906 he was sent to the Epiphan All Saints cemetery church. In 1916 Sergey Molodensky retired from this post of priest and was appointed as a low teacher of the Epiphan teacher’s seminary. After the revolution he became again priest of his former church and stayed in this position till the beginning of an antireligious mass campaign. That time Mikhail’s parents had to leave Epiphan in a hurry. Sergey Mikhailovich had to retire from the order of priest. In the Molodensky family they kept a post card from an unknown but close person who wrote the following:“ All people in Epiphan were shocked and moved to tears when they heard about your departure”. This post card was sent on August 5, 1932.

Nadezhda Mikhailovna (girl’s name Tatevskaya) studied in Tula eparchial women’s school. She passed the 4th and 5th class but when she passed to the 6th class in 1897 she failed; she was given exams for all disciplines. By the same time her father, the priest Mikhail Pavlovich Tatevsky, was transferred from the church of Alexin to the Dimitri church of Tula, where Sergey Mikhailovich was the psalmist.

There are no details in the family about the roots of Mikhail Sergeevich’s parents. The names of Molodensky came from the country name of Molodenky in the Epiphan district (now Kimovsky region).

There were two churches in the big village of Molodenky which exists up to now. The first is a St. Mary Church (built in 1803 instead of a wooden one by the owner of the village, count Kirill Grigorievich Razumovsky) and the second in the honor of the Saint Duke Alexander Nevsky (reconstructed as a stone building in 1882 by Petr Fedorovich Samarin). Until recently one could see the remnants of the first church, the second has been destroyed completely.

In accordance with the book “Russia. The full geographic description of our motherland” of 1902, in the beginning of the XXth century in the village Molodenky there were 900 inhabitants, there was a hospital, a school and a sugar making factory. In the time of the peasants’ liberation the village was owned by Petr Fedorovich Samarin, the brother of the well known peasantry liberator Yuri Fedorovich Samarin. The life at the end of the XIXth century was described by Belgard 1916. Samarin was visited by the Lopukhins, the Obolenskies, by L. N. Tolstoy, S. N. and E. N. Trubetsky, afterwards well–known philosophers.

Among the priests of Tula province in the second part of the 19th century there were some Molodenskies: Ioann (priest in the village of Kishkino, died in 1862), Mikhail (priest in Nikolskoe, died in 1871; most probably he was the great–grandfather of Mikhail Sergeevich). Mikhail Mikhailovich was 1873 ordained priest, being psalmist of the Spasopreobrazhenskaya (Transfiguration of the Saviour) church of Tula, and in 1877 he was ordained second priest in the church of Lutorichy and Kozlova Sloboda. In 1894, information about his death was made public. He probably was the grandfather of Mikhail Sergeevich, In 1901, Mikhail Mikhailovich’s wife Alexandra received a priests pension. Petr Mikchailovich graduated from Tula seminary in 1875; in 1879 he was appointed second priest in the united Rojdestveno–Losinsko–Nivensky parish.

The name Tatevskie originated from the village Tatevo of Tula province. The history of that village was described by Troitsky 1910, member of the Moscow archaeological society and honorary member of the Society for protection and conservation of the art and history memorials in Russia. The name of the village originates from the name of the local landlords who owned the village and the peasants. The family founder was duke Ivan Fedorovich Tath, who was known due to his victory against the tartars in 1547 on Prone. The parish existed since the 12th century. The church of Nikola Thaumaturge was built on the cemetery most probably by duke Alexander Mikhailovich Volkonsky in 1780 who was the owner of the village at that time. This church and the church of the Assumption of the Holy Virgin of the neighboring village Zaitsevo composed one and the same parish. In 1864 Aleksey Tatevsky became a priest in Tatevo but it is doubtful that he was the first to bear that name because in 1862 Mikhail Pavlovich Tatevsky was indicated as the deacon of Osinovaya Gora. In 1875 he — or maybe his namesake — after graduating from the seminary became a priest in Lubikovo and since 1877 in the church of the Holy Virgin of Alexine.

As Alexandra Mikhailovna and Vladimir Mikhailovich (the widow and the cousin of Mikhail Sergeevich) remembered, Mikhail Pavlovich was the grandfather of Mikhail Sergeevich. In the village of Borovkovo of the Tula district in 1862 the priest was Mikhail Semenovich Tatevsky, in 1865 he became the priest of the Nativity of Christ and Nikolo–Zaretsky churches of Tula.

More detailed information about Mikhail Sergeevich backgrounds does not exist.

The goodness of Mikhail Sergeevich’s parents was unusual. They hosted poors, permitting them to spend nights at their house, Sergey Mikhailovich did not take money from the poor for which the neighboring priests criticized Nadezhda Mikhailovna.

For the 175th anniversary of the school in Epiphan which was celebrated in 1990, the local historians headed by the teacher Sergey Vasilievich Kusakin were collecting information about its graduates. The memories of Ksenofontova were dedicated to the history of the school as well. There are memories about the school successes of the Molodenskies. Vladimir organized a circle on mathematics in the school and later he became a chemist. Mikhail had fallen in love with astronomy while he was a child. Once during the dinner he tried to start to talk about the sun's rotation. The oldest started to laugh at him. He began to cry and left the room without eating. When he was older he was in charge of the literature–critical section of the school magazine "Blasts of Ideas". By that time he liked Pisarev and followed his ideas. This work was supervised by Klavdia Gavrilovna Savelieva. The love for physics and mathematics was raised by the teachers Nikolay Fedorovich Jilich and Dmitry Alexandrovich Masing. As Mikhail Sergeevich recalled, they developed an interest in research in the exact sciences. Thanks to Kusakin we have data about Jilich and Masing. The first was Polish (may be an officer of the tsarist army); when and how he appeared in Epiphan nobody remembers. In the twenties in Epiphan there existed two schools who were called by the names of their directors "School of Bulgakov" and "School of Jilich". Most probably Mikhail Sergeevich graduated from the "School of Jilich". At that time Nikolay Fedorovich worked at the secondary school and was the class leader of the first who graduated in the 10th class of 1936. In 1938 together with some other teachers he was removed but soon came back and worked in the Epiphan school till his death in 1957.

The pupils and colleagues called Masing a great teacher. Dmitry Alexandrovich was a graduate from Moscow University and started his pedagogical activities in the country school of the village Podjom of the Tambov province. Having noted the talented pupil Vasya, the son of a poor peasant Mikhail Solokhin, he asked the father to give him his son for education and supervision without any compensation from the part of the parents. Vasily graduated with success from the geographic faculty of Moscow University and was assigned to work in Epiphan where by that time the inspector of the schools had become Masing. Masing had some pupils under his care and he helped some students financially.

After the revolution in the twenties, Masing was teaching mathematics and physics at the Epiphan secondary school. Then he started to work at the Epiphan teaching school, also offering workshops for the teachers of mathematics and physics of the Epiphan region. Kostomarov, who lived in the family of the adopted son of Masing, Vasily Solokhin, remembers: "On his slim tall figure he was wearing the dark blue coat of the Ministry of Education, with a double row of buttons ... On the badges he had double–headed eagles ... His coat was not buttoned accurately and the coat–tails, being of different height, looked like the wings of a big wounded bird ... On the big face he wore spectacles with thick glasses. His beard stuck out, and the head was surrounded by silver hair like an aureole. He was looking straight in front, giving the appearance that if you did not give him the way he would step into you straight away ...". Masing died in the fifties in his sleep, unmarried for the whole life. In 1992, Kostomarov published his memories about Jilich and Masing.

The work of outstanding teachers gave excellent results: the first graduations of the twenties brought plenty of bright heads. These were of the same age: Yury Mikhailovich Piatin, doctor of technical sciences, Ilya Mikhailovich Rodin, doctor of biological sciences, Mikhail Petrovich Chumakov, a well–known physician and fighter against poliomyelitis, and the brothers Molodensky. Sergey Sergeevich Molodensky was a famous teacher of physics in Leningrad.

Epiphan at present is a very small town. The center of the region is the new town of Kimovsk, and life stopped in the old Russian town. Epiphan appeared in 1571 as a guarding point. During the government of Ivan the Terrible it was mentioned together with other such points on the rivers Don and Mecha. Epiphan had a wooden castle with nine stone towers, one of which survived. Since 1777 Epiphan became a town in the Tula district. Agriculture started to develop in this district. Mikhail

Sergeevich Molodensky often remembered the nature of his native land. The commerce also developed. Many well–known people had their houses here in Epiphan: the Bobrinskies, Golitsiny, Dolgorukies, Samarinies, Raevskies, Shahovskies etc. Well known were the merchants Ovodovy and Rastorguevy. The writer G. I. Uspensky was teaching in Epiphan about half a year. In the beginning of the century it was a small country town with a population of some 10000 people. The town became full of life specially during the fairs, which were held at the Market Square near All Saints church where Sergey Mikhailovich Molodensky served; his family lived nearby at Novoslobodskaya street. The merchants came not only from different parts of Tula province but from neighboring provinces as well. In his childhood Mikhail Sergeevich was watching all that. Now All Saints church is destroyed. A distillery was built from its bricks.

In 1923, 14–years old Mikhail had finished seven years of school and started to live in the family of his uncle — doctor Mikhail Mikhailovich Tatevsky in Tula, where he studied in the railway school. After finishing it he worked for about a year as accountant in the forestry at Yasnaya Poliana. Having earned some money there for his work, Mikhail Sergeevich went to Leningrad where his oldest brothers lived, and he passed the entrance examinations for the Pedagogical Institute named after Hertsen, but for the son of a priest the way to the highest education was prohibited. For Mikhail Sergeevich this was the end of all hopes and plans. However he managed to get into the astronomical section of the physical–mathematical faculty of Moscow University disguised as an exchange student. The dean’s office was not happy with such a situation: they did not give him lodging in the student dormitories and even tried to put obstacles to his presence at the laboratory work. Help came from a relative from the mother’s side, Alexey Vladimirovich Zhdanov. He had a room in a common apartment where he hosted Mikhail Sergeevich till he graduated from the University. But Mikhail Sergeevich used any possibility to live with the students and sometimes he passed the night at the conference hall of the observatory of the University at Presnia. The sister of Molodensky’s colleague Vladimir Fedorovich Eremeev remembers a poorly dressed boy who showed up at their place. Also the nephew of Eremeev remembers him. Mikhail Sergeevich lived with the Eremeevs occasionally, in particular during the war, splitting wood and once helping in calculations for their home planetary. The Eremeev family was not rich either, they did not have money for the train and Vladimir Fedorovich had to walk to the university from Blagusha, a small village near Izmailovo. They kept their friendship for the whole life.

Coming once back to the common dormitory, Alexandra Mikhailovna Mameshina, an aspirant of the State Astronomical Institute named after Sternberg (or Shternberg), found Molodensky in her bed with a very strong pneumonia. Mikhail Sergeevich had got a heavy chill while determining the temperature coefficients of pendulums (he was helping L. V. Sorokin). In this way Molodensky recovered from the illness in the bed of his future wife in the women’s common dormitory of Moscow University on Trifonovskaya Street, and she slept on the floor nearby. Mikhail Petrovich Chumakov, his friend and fellow countryman found and helped him. The conditions of study in those times was described by Heifets (1992) giving characteristics of the style of life. (This article is published in the present volume with the author’s additions.)

By the time when Mikhail Sergeevich became a student, the first Soviet period of the comprehensive research of the Kursk Magnetic Anomaly (development of the methods and instruments for the geophysical field work including gravimetrical observations) was finished. By the same time, Krasovsky had finished planning the future astronomic–geodetic network of the Soviet Union, and in Holland, Vening Meinesz had obtained the formula for calculating the components of the deviation of the vertical based on a gravimetric survey. Without necessary confirmations, Vening Meinesz stated that it was sufficient to know the anomalies of the gravity in a limited surrounding area, which gave a reason for starting the practical works.

All these circumstances prepared for the beginning of the planned geodetic and geological research of the whole country, contributing to the development of geophysical methods of reconnaissance of natural resources. Teachers and scientists of a new type become necessary. On the physical–mathematical faculty of the Moscow University, a specialization in geophysics was introduced. Geodetic problems entered the program of works of the University astronomic observatory since its foundation in the beginning of the 19th century (Blazhko 1940). The deep interest for geodesy of the mechanics professor F. A. Sludsky was not accidental. At the University, an astronomic–geodetic institute was created, so there was a good base for developing the preparation on higher geodesy and gravimetry. There was also a desire to replace the astronomic specialization by a specialization “geodesy and gravimetry”. To such a group, Mikhail Sergeevich was admitted. M. S. Zverev left his memories about this period (1959); he studied with Molodensky in the same group. The courses for gravimetry, theory of the figure of the Earth and spheroidal geodesy were given by A. A. Mikhailov, triangulation was given by M. D. Soloviev, and a course on grade measurements was given by F. N. Krasovsky, specially invited by the students. Astronomical courses were given by S. N. Blazhko and S. A. Kazakov, mathematical analysis by N. N. Luzin and A. Ya. Khinchin, theory of differential equations by V. V. Stepanov, a course on numerical mathematics by S. A. Kazakov, differential geometry and theory of surfaces by S. P. Finikov. Beskin published his memories about the Moscow physmat faculty of the beginning of the twenties (1993).

M. S. Zverev in the above–mentioned memories writes about the extremely respectful relationship of the group of students and especially of Molodensky with Feodosy Nikolaevich Krasovsky. A. A. Mikhailov in those years worked translating the well–known monograph of P. Pizzetti (1913), and Mikhail Sergeevich remembered with pleasure how they checked the results and corrected plenty of misprints in the original text. In this way in 1933 the Russian translation became much better than the Italian original. And this was not surprising because in Italy they do not know that monograph so well as in Russia ...

His creative urge asserted itself: already as a student Molodensky improved the method of Pevtsov’s pairs for determination of latitudes and published an article about it in the “Astronomical Journal” (1931). In January 1931 S. N. Blazhko, the favorite teacher of Mikhail Sergeevich, invited him to work in the Astronomic–geodetic scientific–research Institute (AGNII) of Moscow University. Soon this Institute, as well as the Astronomic Observatory of Moscow University, the State Astrophysics Institute and the Astrophysical Observatory in Kuchino were put all together in the Astronomic Institute named after P. K. Sternberg. Molodensky was transferred to Kuchino to V. G. Fesenkov. There he was obliged to work on photometry of black nebulae. F. N. Krasovsky and I. A. Kazansky invited Molodensky to the gravimetrical section of the geodetic institute, which was soon reorganized in the TsNIIGAiK. In Kuchino he finally found a room in a communal apartment. In this room with him there also lived his father, and his mother started to live with daughter Elena in Tula. But the quiet life did not last long: during a meeting dedicated to the Party cleaning the “community” became aggressive on the fact that a former priest lived in the apartment of a scientist. N. D. Moiseev managed to support Molodensky only morally, Fesenkov did not manage to support him at all.

Soon there was a possibility to be transferred for the main work to TsNIIGAiK, which by that time was located in Leningrad. Mikhail Sergeevich and Alexandra Mikhailovna got married in 1933 and lived in a Leningrad hotel paid by the Institute. After some years TsNIIGAiK came back to Moscow and Molodensky's family got a room in a barrack at Potylikha. Till the war his father lived with them but during the war he died in Tula.

In 1932 by the decree of the Soviet of Labor and Defense the principles for realization of the state gravimetric survey were defined, and Mikhail Sergeevich at once became one of the authors of the first "Instruction on the gravimetric works for the general gravimetric (pendulum) survey of the USSR" (1935). One year before in 1934 there was published his work about the co–swinging of the support during the swinging of a pair of pendulums with different amplitudes and arbitrary phases. There is a letter of Molodensky to V. F. Eremeev dated 1935 from Leningrad: the first attempt of computing the corrections for the co–swinging led to an improvement of coincidence and a twofold decrease of the differences of the mean periods. The reality of the corrections and the necessity of their taking into account were established, and the method of calculation was simplified.

Besides the practical and theoretical works on the pendulum survey and extremely important works on geodetic theory, Molodensky started absolutely new work in 1938 for the creation of a new gravimeter (theory, construction, technology of production, field tests). In the personal records of Molodensky there is a writing made by himself: "From 1938 till 1941 I had to investigate questions of the gravimetric instrumentation". The words "I had to" appeared most probable because his soul aspired to theoretical works. But the gravimeter was urgently necessary, and Molodensky created a system of his own. So in our country even during World War II there appeared the first spring ring gravimeter of Molodensky (GKM). After the war Molodensky took part in the creation of the ring astatized gravimeter GKA based on the GKM. The instrumental and methodical works of Molodensky are described in more detail by Heifets.

In 1951 Molodensky was given, for the second time, the State Award for the creation of the GKA, together with colleagues. Earlier in 1946 he had received an award for his monograph "Basic problems of geodetic gravimetry" (Works of TsNIIGAiK, 1945, N 42). The work was defended by the author in MIIGAiK as a dissertation for the degree of doctor of technical sciences (the degree of candidate he had received in 1938 without defending). One of his opponents — N. N. Pariysky — stated in all fairness that each of the five parts itself deserved a doctor's degree. And the opponent Mikhailov noted that the work transcended their time, opened a new page in geodesy, and would be completely appreciated only in 50 years. The work was indeed outstanding and in one year its author was elected corresponding member of the Academy of Sciences of the USSR.

During the War Mikhail Sergeevich had an additional employment as lecturer at the chair of gravimetry and geophysics of MIIGAiK (lectures and seminars for students and aspirants). Not long before leaving the faculty in 1947 he proposed to radically strengthen the preparation of the students of astronomic–geodetic specialization in physics and mathematics but was not supported.

From 1943 until 1956 Molodensky was chief of the laboratory of gravimetry of TsNIIGAiK. In 1946, as an additional duty, he became chief of section of gravimetry in the Geophysical Institute of the Academy of Sciences of USSR. Since 1956, this institute, already transferred to the Institute of Earth Physics (IEP), became the main place of work of Mikhail Sergeevich. In 1956 he, though not being a Party member, following the order of the Party Central Committee, became the Director of that Institute, in spite of the fact that such a work did not fit with his character. In the next year it caused a severe illness which he did not manage to overcome. Nevertheless he went on with intensive work, in particular on geodetic themes and since 1960 — without being paid for it. Molodensky's researches involved the main theoretical parts of geodesy. They are represented in the monograph "Methods for study of the external gravitational field and figure of the Earth" (TsNIIGAiK 1960), in preparation of this book he was helped by Eremeev and Yurkina. This work now serves as a base on which the modern geodesy develops. For the research described in this book and for the work made on the research field of IEP on the elastic tides and free Earth nutation, in 1963 Molodensky received the Lenin Prize.

To understand correctly the place of Molodensky's works in geodesy it is necessary to recall the main stages of its development for the last two hundred years.

Grade measurements permit, as it is known, approximately to determine the mean curvature of the Earth level surface in the area of arcs under research. Applying this method in Europe in the second part of the 18th century and the beginning of the 19th century, demonstrated that the curvatures of the arcs under research do not change in the way as it had to be on the level surface of an ellipsoid. The gravimetric observations showed the non–constancy of gravity along the parallels and a change along meridians different from its previous incorrect computation. In particular this was shown by geodetic, astronomic and gravimetric observations of Biot and Arago made as continuation of the French grade measurements of the 18th century. This made up the additional volume of materials "The bases of the metrical system" (Recueil 1821). As the result of these experimental facts, in geodesy there appeared a new object for research: the "Earth surface in a mathematical sense" (Gauss 1823), which later was called Geoid. The necessity of geoid research seemed obvious because it corresponded to the universal but not always well–recognized fact that topographic surveys determine the visible Earth surface relatively to a unique level surface, the sea level (sphere or ellipsoid earlier and now geoid). Without undue simplification we can state that all the efforts of geodesy of the next century were directed to improving triangular and gravimetric methods of research, not of the Earth but of the geoid.

The most important theoretical work on the geoid determination on the base of gravimetrical data is due to G. G. Stokes (1849). Generalizing the theory of A. C. Clairaut, Stokes proved that for the theoretical research of the planet's surface, what is important is not the state of the hydrostatic equilibrium, but the constancy of the gravity potential on a level surface, which can have not only ellipsoidal but an arbitrary form as well. In the less general form with a precision up to the first terms of the expansion into series of spherical functions, this fact was known to P. S. Laplace 1785. The Stokes theory was original indeed: to the whole mass of the planet, potential of the centrifugal force and external level surface of the planet there corresponds uniquely the gravity potential in the external space and accordingly the only possible distribution of gravity at this surface. The methods of the calculation of the gravity potential and the gravity at the known geoid surface are based, as we use to say now, on the solution of the first external boundary problem of potential theory. It was taken into consideration that, for geodesy, it is necessary to solve an inverse problem: the determination of the geoid on the base of the gravity on it. Stokes also solved this supposing that the geoid is close to the level surface of a chosen rotational ellipsoid of small flattening, so that it became possible to determine the disturbing potential and together with it the small heights of the geoid above the ellipsoid. In such a case, the determination of the geoid reduces to the solution of a third external boundary problem and the solution is the known series and integral formula of Stokes.

Stokes' solution was based on two important assumptions: the first that there are no attracting masses outside the geoid, and the second that the gravity must be measured on the geoid itself. In such a way, Stokes' theory confronted geodesy with two problems: the problem of the regularization of the geoid which makes the geoid an external level surface, and the problem of reduction of the gravity measured at the Earth surface and corrected for the regularization to the surface of the regularized geoid. Indeed, Stokes himself gave the approximate solution of these tasks but an adequate understanding for them appeared only by the end of the 19th century.

The main method of geoid research in the 19th and 20th centuries was the method of the astronomic–geodetic networks, the grade measurements. The task of reducing the measured horizontal angles, baselines, astronomic longitudes and latitudes to the geoid was supposed to be simple enough. The measured horizontal angles were considered equal to the corresponding angles at the geoid and the problem at that time was only to reduce baselines and astronomic longitudes and latitudes. It was considered that both reductions could be determined with sufficient accuracy by the heights of the measurement points above the geoid (by orthometric heights), and that these heights could be determined accurately enough by leveling. The problem of regularization of the geoid was not studied in its generality, but only sometimes in connection with some peculiar question, for example, the determination of the orthometric corrections for leveling. All this led to the fact that the question of the reduction of gravity did not play an important role in geodesy. In fact, it was studied separately from the reduction of all the complex of astronomic and triangulation measurements, receiving rather the character of tasks of geophysics (the works of J. H. Pratt, G. B. Airy etc.).

The interest for the reduction problem in its full generality increased at the beginning of the thirties, when a possibility arose in our country for a practical use of gravimetric deviations of the vertical in the astronomic–geodetic networks, and their insufficient precision especially in the mountains was soon detected. Following the initiative of Mikhailov and his general indications, a number of generalizations of the Stokes theory were published for the case when the real geoid of the Earth is to be determined: the non–regularized geoid (Molodensky 1948b). Solutions of this problem were proposed by Moiseev (1933, 1934, 1935), Malkin (1934, 1935), and Molodensky in two articles in 1936.

The attempts to improve Stokes' theory did not give any noticeable practical results: the problem of reducing the measured values to the geoid was not solved. About this, Krasovsky wrote (1944) the following: "In the difficult problem of the reduction of gravity, it is so far possible to mention only few achievements, in spite of the fact of the great importance of the researches as such. As we think, here even bright mathematical approaches do not give expected results because of the failure to take into account geophysical aspects of the problem. "

Molodensky in his monograph of 1945 introduced some additions to the results of Moiseev. It was made easier to compare all the three researches, it was made possible to interpret exactly all the variants of the theoretical solution of the problem of regularization and reductions and to compare them concretely with the method of condensation modified by A. A. Mikhailov in 1940 and 1945. It became obvious that to determine the geoid it was necessary to know not only the gravity on it but in addition that part of gravity which was caused by the attraction of the continental masses. Molodensky 1945 wrote in his monograph (p. 30): "A rigorous solution is possible if we know the density, at each point, of all masses located outside the geoid". In his article of 1948 (p. 194) he added: "Even if all geological data are known, a sufficiently precise reduction to the geoid is related to the solution of a hard problem of potential theory, as the reduction is to be made to the unknown surface of the geoid from the very complicated and also unknown physical surface of the Earth, on which the boundary values are being determined immediately by the observations".

Many geodesists did not take seriously such a dependent position of geodesy from geology; they even considered that it was good, because geodesy was entering geology, and that means that it is necessary to build a bridge between the two sciences. In such a way, for example, Krasovsky (1944) was thinking. This was also supported by historic examples — geodesy required geology for isostatic compensation. Simultaneously one studied the joint use of different data for concrete tasks. Magnitsky used the geoid heights, determined in the astronomic–geodetic networks, for determining local influences in the gravity anomalies (1945).

In such a situation in 1945, there appeared the article of Molodensky "The role of geophysics and geology in the study of the figure of the Earth". In this article the author stated that when a gravimetric survey is completed, the additional astronomic–geodetic network data or other data on the external gravity field did not produce useful information on the masses needed for reduction. The geophysical researches of Pratt, Airy etc. type could be interesting for geodesists from the point of view of explanation of the results obtained by them, but they cannot be taken into account for research on the Earth's figure. All the geodetic measurements are being made on the Earth surface. There are no attracting masses outside this surface, but the gravity potential changes on it, which raises important difficulties. That is why one cannot use the Stokes theory immediately. It is necessary to generalize it for the following case: to use the Earth surface as the boundary for determining the disturbing potential. Physical geodesy must become analytical. Molodensky's monograph (1945), in its third part, precisely stated: the task of geodesy is to determine the external gravitational field and the surface of the Earth. With this change of the task of geodesy the author had to specify the new concepts which replaced or generalized the old ones. There appeared the normal (auxiliary) heights, very close to orthometric heights but capable of being calculated from the measured data only. The height anomalies which are fully consistent with the normal heights, are determined by the external disturbing potential. They play the role of the geoid heights, but relate to the Earth surface and indirectly even to the whole external space. The anomalies of the gravity did not change formally (according the method of calculation), but this happened because the anomalies were calculated on the geoid only with the free air reduction, i. e. too simply, without taking into account the anomalies of the vertical gradient of the real gravity. Now, in fact, the gravity anomalies have to be determined at the Earth surface and it is necessary to reduce not real but normal gravity. For the purpose, however, the free air reduction became necessary and sufficient!

The deviations of the vertical were calculated in an analogous way: it is necessary not to reduce the direction of the real force line to the geoid for comparison with the direction of the normal to the ellipsoid, but the direction of the tangent to the force line at the very point of measurement must be compared with the direction of the normal to the ellipsoid or with the direction of normal gravity. The difference of these two methods of calculation of the meridional component of the deviation of the vertical corresponds to the well–known Gaussian correction of 0",171H sin$2B$.

It was customary to consider that this approximately determines the curvature of the real force line and approximately reduces the observed astronomical latitude of the measurement point to the geoid. But it happens in Molodensky's theory that Gauss' correction is necessary for comparing the direction of gravity with the normal gravity direction at the same point. And in general it is impossible anyway to reduce measurement values because the required additional information is usually unknown in principle. To compare the measured values with the normal ones, the latter must be calculated at the places of the measurement. From the new view of reduction it followed, that the boundary condition for the disturbing potential had to be related not to the geoid surface but to the real Earth surface, where all the geodetic measurements are being made. This is why even the disturbing potential can be determined only outside the Earth surface or on it. It is possible to solve the problem again in a linear approximation, not at the geoid as it was made by Stokes, but now at the Earth surface. When determining the disturbing potential, there again appears the third external boundary problem, but now with an oblique derivative. In the boundary conditions of Stokes and Molodensky there is the derivative of the disturbing potential with respect to the line of force, but in Stokes' case the plumb–line strictly coincides with the normal to the boundary surface (geoid), and in Molodensky's case there appears a difficulty: the force line does not coincide with the normal to the new boundary surface (Earth surface). That is why the vertical derivative of the disturbing potential becomes oblique relatively to the boundary surface. The necessity of taking this into account leads to the appearance of the derivative not only with respect to the ellipsoidal normal, which is simple to represent through the gravity anomaly and the disturbing potential, but to the appearance also of the two derivatives in directions orthogonal to the ellipsoidal normal: the two components of the deviation of the vertical. It is very ingenious how the author was able to eliminate these terms under the integral sign and found, exactly in the linear approximation, a linear integral equation which contains only one unknown: the disturbing potential at the Earth surface, and the term on the right–hand side depends on the gravity anomalies only. The solution of the integral equation determines both the Earth surface and the external gravity field, as well as all other quantities of interest to the geodesist.

By the way, we should mention one detail of Molodensky's theory which goes, if we can say so, beyond the limits of geodesy and is of general mathematical interest. The integral equation referring to the unknown Earth surface is transferred by the author to the surface determined by the normal heights, which is now called telluroid. This caused an error in the result which was estimated by him and proved negligible within the frame of the linear problem. Now the operation of transferring from the Earth surface to the telluroid can be practically omitted. For this it is enough to use the space triangulation points, the heights of which above the ellipsoid are determined very exactly, and in some points one must add, to the normal heights, the height anomalies, known as usual with an error not greater than 3 m. In this case the slowly changing errors of the height may be neglected.

These errors will be rather unimportant for solving the boundary problem even in the future. Nevertheless the problem exists and its research attracted the attention of the mathematicians Krarup (1969), Hörmander (1976), and Sanso (1978).

In their works from the point of view of a geodesist the importance of non–linearity of the Molodensky problem was overestimated. The good precision of the Earth surface determination by leveling and Stokes' formula, was not taken into account, not to speak of space triangulation and GPS. But from the mathematical point of view also Giraud's theory (1939) was not taken into consideration, which not only proves the existence of the solution, but also expresses it in an explicit form by means of a single layer on the Earth surface. The solution of the problem will be obtained by solution of the corresponding integral equation. The non–linearity of the problem plays the less a role, the less is the order of the gravitational potential derivatives, which it is to be determined. A certain role is played by Sagitov's (1983) connection between the detailed knowledge of the density distribution and the orders of the potential derivatives. Giraud's theory supposes a continuous distribution of data on the boundary surface, but the gravimetric observations are discrete. Thus the survey has to be made with such density of stations that the results of the measurements can be interpolated between the gravimetric points. The connections between the survey density and the resulting precision, taking into account the terrain characteristics, are well studied, so that within the limits of the required precision it is possible to consider the initial data as continuous.

The book of Kupradze et al. 1979 contains a criticism of Hörmander's approach. In the most precise solution the single layer it is to be distributed not on the telluroid, but on the Earth surface. Then the corrections for non–linearity of the solution may be determined by a method of Newton–Kantorovich (Kantorovich and Akilov 1982). More details on these questions were given by Yurkina (1981). But let us come back to Molodensky's works on theoretical geodesy.

In the article (1948a) he obtained a new and simpler integral equation for a single layer density at the Earth surface, the potential of which outside the Earth is equal to the disturbing potential. Investigating this equation, Molodensky came to the conclusion that a particular solution exists for a spherical reference field, if the initial data do not contain errors. The coordinates of the mass center of the Earth and the gravity potential at the origin of heights enter into the general solution as unknowns. Those four additional unknowns, as it was described in the book of 1960, may be determined on the base of a gravimetric survey of the whole Earth and a wide astronomic–geodetic network. The last unknown which earlier was solved by grade measurements, now, together with the world gravimetric survey, received a new sense. The old method of grade measurement combined with an additional local gravimetric survey is now being solved very precisely by astronomic–gravimetric leveling.

The above–mentioned condition of errorless gravity anomalies does not depend on the ellipsoidal reference field. That is, a particular solution of the task exists even if the given gravity anomalies are arbitrary (Eremeev, Yurkina 1972), and in this way the requirement of absence, at the sphere, of the harmonic of the first degree in the expansion of the gravity anomalies is not necessary.

In the following years Molodensky proposed some solutions of the equation of 1948 and in his monograph of 1960 exposed a very general effective method which found its practical use in other solutions of the boundary problem (Brovar 1993). The theoretical research described here comprises studies and solutions of geodesy's main task, Molodensky's problem. It is possible to formulate it in the following way: if gravity and differences of its potential are measured at the Earth surface, to determine the Earth surface and its external gravity potential, if also the potential of the centrifugal force and the Earth mass or the distance between two distant points are known (Brovar, Magnitsky, Shimbirev 1961). It is proved that the solution of the problem always exists and is unique. The mathematicians (Guenther 1987, Jorge 1987, Danilov 1996) confirmed the correctness of the solution, including the possibility of the corrections in a linear approximation.

The investigation of the main problem of geodesy leads to a natural unification of Molodensky's problem with the first and second boundary problems into a more general problem (similar to the unification of the direct problem with the inverse problem), analogous to a relation between the solutions of the three boundary problems of mathematical physics.

It is possible to change the initial data in Molodensky's problem in keeping with the needs of practice. In a joint solution for different parts of the surface it is possible to use boundary data of different types: mixed anomalies at the ground and usual anomalies at the sea surface. To each type of data there corresponds its own type of integral equations, and it is possible to solve them jointly. The possibility of such a mixed problem solution was shown by Vishik and Ladyzhenskaya in 1956. The satellite altimetry data also are included into the number of boundary data. It is possible to solve overdetermined problems. The results of the laser satellite and GPS tracking can replace the grade measurement data. In this way, modern geodesy problems received a mathematical foundation and took an appropriate place in mathematical physics.

In the thirties, besides the necessity to solve exactly the problems of theoretical geodesy, Soviet geodesy was also faced with other urgent tasks, caused by the huge extension of our astronomic–geodetic network. With the advance of the triangulation through Ural, it became clear that the practical use of the ellipsoid of Bessel, which was used as the reference ellipsoid with orientation in Pulkovo, was no longer adequate. Izotov, under guidance of Krasovsky, began to determine the semi–major axis and flattening of a new ellipsoid, which as far as possible could be used for the whole Earth. A systematic study of the Soviet and foreign triangulation material was started. In 1940 the dimension and the flattening of the Earth ellipsoid were determined.

According to the opinion of Feodosy Nikolaevich, another source of errors was the method of translation used in processing the astronomic–geodetic network. Krasovsky indicated the necessity of passing to the method of projection, involving additional reductions of the baselines from geoid to ellipsoid. The most detailed account can be found in Krasovsky's book 1942. According to the calculations of Feodosy Nikolaevich, the precision of the astronomic leveling was not satisfactory for this purpose. The errors of the translation method were insignificant in small territories where the necessary precision could be obtained by diminishing the distances between astronomic points. This was impossible in our country. The possibility to determine the vertical deviations by gravimetric data with sufficient accuracy was made clear by gravimetrists of the TsNIIGAiK, a method of calculation was worked out, and Krasovsky gave to Molodensky the task to improve the precision of astronomic leveling without increasing the total number of astronomic observations, by a new use of the gravimetric data. Kazansky and Eremeev remembered that first Feodosy Nikolaevich considered the possibility to solve the problem with the use of isostatic reductions. Later he began to give more importance to the use of gravimetric data in astronomic–geodetic works in general and to astronomic leveling in particular. One can follow the change of his views in his publications. He started to consider the idea of astronomic leveling made more precise with gravimetric data instead of using topographic–isostatic reductions in 1934. And in 1935 Krasovsky wrote: "Fortunately we have competent and talented workers to go on in this direction ..." In 1936 he already described briefly the so–called bifocused grid and the principle of astronomic–gravimetric leveling. A detailed description of the ideas underlying the method: evaluation of the minimal radius of an area of gravity anomalies and calculation of the grid, was published by Molodensky in 1937. At the end of his article the author wrote: "The work presented here was made in the Scientific Research Institute of geodesy, cartography and air survey in 1935 under the general supervision of Professor Krasovsky, by whose initiative it was included into the science–research plan of the Institute, and Professor Kazansky, the leader of the astronomic–gravimetric section of the Institute". The institute was soon named Central Scientific Research Institute of Geodesy, Aerosurvey and Cartography (TsNIIGAiK). It is now clear whom Krasovsky had indeed a competent and talented coworker. The article about the first practical realization of astronomic–gravimetric leveling, made together with Lozinskaya, was published in 1939. In 1939, the work was continued by B. V. Dubovsky in a more extensive way, involving significant territory, and permitting to reduce the baselines for the ellipsoid of Bessel more precisely and to pass on to the new processing of the astronomic–geodetic network.

In 1943 the astronomic–gravimetric leveling was repeated on the new ellipsoid of Krasovsky with the previous orientation in Pulkovo. The reduction of the baselines to this ellipsoid permitted first to eliminate systematic influences of the error in Bessel's ellipsoid and second to perform the new network adjustment with the new optimal orientation of the "reference–ellipsoid of Krasovsky" (Izotov 1948).

In 1944 Molodensky paid attention to another profound difference of the two methods of the processing of astronomic–geodetic data. In the method of translation of the triangulation, an incomplete and consequently incorrect reduction of the baselines (as if it was for the geoid instead of the ellipsoid) leads to incorrect calculation of the lengths, and this method forces us to lay out ("translate") these lengths at the ellipsoidal surface. Thus the calculated differences of geodetic coordinates are systematically distorted. Astronomic latitudes and longitudes and calculated geodetic coordinates result in being related to different points of the ellipsoid and do not characterize the angle between the plumb–line and the ellipsoidal normal. Molodensky detected the connection between the coordinates obtained by these two methods: the connection between "mixed" and "proper" deviations of the plumb–line. As an example, the corrections of coordinates were calculated along a parallel, permitting the passage from one method to the other. The TsNIIGAiK's workers were indeed shocked: the readjustment of the network, which is very difficult to perform, was in a way replaced by an integral along a parallel. Differences in longitudes at almost all the closer points did not exceed 0,04", but numerous big differences were related to points distant from the chosen parallel. In 1959 Irene Fischer described the use of this method for correcting the results of network translation along the giant arcs from Canada to Chile and from Scandinavia to South Africa. The corrections described are small within an angular distance of 30^0 from the starting point, then they start to rise quickly and finally reach the order of the values under determination: geodetic coordinates in the method of translation become poorer than astronomic coordinates. In the astronomic leveling of America over a distance of 40^0 (from Meades Ranch to equator), corrections were on the order of some meters, but in the Southern hemisphere at 30° of southern latitude they reach up to 40 meters, and at –40° already 60 meters. In Africa at the latitude – $-34°$ the correction reaches up to 100 meters for an arc beginning at the latitude $+65^{\circ}$.

Molodensky's corrections were described in the book of Pellinen 1978 and researches were continued in his article 1983. It was proved that by calculation of the heights of the quasigeoid in large territories the indirect effects could exceed the influence of the direct effects. The detected dependencies were used in our country during processing each link of triangulation. Besides the longitudinal displacement he took into account also the transverse displacement caused by distortion of Laplace azimuths due to incorrect geodetic longitude.

During many years, Mikhail Sergeevich Molodensky was occupied with different questions of geometric research of the Earth surface. New proposals were described in the work in 1954. In its entirety, this direction was presented in the book 1960. In this work there was important its direct connection with the general new view upon the tasks of geodesy: the necessity of research of the Earth surface in space and consequently the rejection of solving tasks on the ellipsoid surface with the help of geodesic lines. This new view of the tasks of geodesy made somewhat more difficult the passage from the horizontal angles measured in triangulations to angles between the straight lines connecting the points, but this difficulty has a computational character only. The new approach permitted to reject, in spheroidal geodesy, in principle the methods of differential geometry and to use simple methods of analytical geometry of coordinates transformation in space. The essence of the method consists in using the spatial geodetic coordinates. Exact formulas, containing only elementary functions and suitable for any distances, appear instead of elliptical integrals or series otherwise used in geodesy. Preliminary reductions to the ellipsoid can be used as well, but only as a convenient computational method. Such an approach is connected logically with the geometrical methods of space geodesy. However, colleagues did not in general adopt the passage to chords. But already in the degree measurements from Dunkerque to Barcelona, which served as the base for establishment of the meter (at the end of the 18^{th} century), Delambre performed his computations in terms of spherical triangles, using a theorem of Legendre 1789, as well as in terms of plane triangles of chords between ellipsoidal surface points. In the last case it was necessary to use special tables. The simplicity of Legendre's theorem, his support by Gauss (1841) and the Clairaut–Gauss theory of geodesic lines, developed in Gauss' differential geometry of surfaces (supporting the role of personality in history!) replaced the use of chords, complicating the computation of triangulation and hence the bases of geodetic education during more then 150 years. The theorem of sines, proved by Grunert (1855), for the triangle of chords (the measured angles must be reduced by a fourth part of the spherical excess), very analogous to Legendre's theorem, did not bring back the interest in chords. In Russia the chords were so well forgotten that the announcement of Milan Burša about Grunert's theorem was met with surprise. Now no one could doubt the correctness of Molodensky's approach.

All we said about Molodensky's works in geodesy involves, most probably, the principal but not all the content of his researches. Some works have a more particular character in spite of the fact that they are related to almost all the fields of modern geodesy and their particularity is relative. Now it is necessary at least to recall them and to say a little about their use and further development.

1. By a unique new method all the spherical boundary problems which can find their use in geodesy were solved. Errors of certain previous solutions were eliminated. One of the new formulas is used to express the gravity anomaly through the geoid heights (in practice through the measurable heights of the sea surface above the reference ellipsoid). This formula was published almost simultaneously by Magnitsky and Molodensky in 1945. As it was mentioned by Magnitsky, Rao in 1935 expressed the solution of this problem by a divergent integral. The other new formula of Molodensky expresses the geoid height through the deviation of the vertical and solves a problem posed by Callandreau in 1901. The solution of Callandreau himself was expressed by a divergent integral.
2. The theory and the practice of astronomic–gravimetric leveling were modernized (Ostach 1970, 1994), the field of taking the anomalies into account was reduced.

3. Mistakes in researches of de Graaff–Hunter were shown. A rigorous method of evaluating the influence of missing gravity anomalies in the distant zones on disturbing potential and deviation of the vertical was elaborated. The method is based on acceleration of convergence of Stokes series. Now those formulas serve for calculating the influence of distant zones on these quantities. They simplify and accelerate the computation, permitting to divide naturally ground and satellite information. Numerous modifications were obtained, considerable literature was published in Russia and abroad, and they are used everywhere.
4. The solution of Stokes' problem for the ellipsoid was given with a relative error on the order of the square of the flattening (1956). The solution laid the base for numerous modern methodical achievements. It was used for finding new solutions for creating a more precise theory of Earth surface research, as well as for the solution of other geodetic questions connected with taking into account the Earth's flattening.
5. The investigation of errors in gravimetric surveys, ways of diminishing them through indirect interpolation considering the influence of topography; the influence of errors of a survey and its effect on the precision of calculations of deviations of the vertical (1945). These researches which were initiated by de Graaff–Hunter (1935), in the following years received a significant development and were particularly improved by using statistical methods of processing of observations.

Sometimes one can meet the opinion that Molodensky's theory consists in the solution of a boundary problem or even in the solution of an integral equation. Of course this impression is not correct. It shows that only some sides of Molodensky's works were considered; their fullness, the unity of the created theory and what is most important, the novelty of the ideas are missed. Molodensky's theory is a new theory of geodesy but not an addition to a previous one. Clairaut's geodetic theory starts from the assumption that the Earth surface depends on one parameter (the Earth's flattening) and establishes its connection with the change of gravity at that surface. Stokes' theory rejects Clairaut's hypothesis and sees its task in determining the geoid as a replacement of the Earth surface. Molodensky's theory, however, determines this very surface. It put in front of geodesy a new aim, and the methods of solution are based only on two suppositions: the correctness of Newton's law and absence of change in time of all initial data.

Of course until 1945 in geodesy there existed the understanding that there were some mistakes in the theory (uncertainty of observed astronomic latitudes and longitudes, reduction to the geoid, questionable methods for calculation of orthometric corrections in the leveling networks, reduction of baselines etc.), but all of them were studied separately and were not considered too important. For the practice of those times such view at "small" theoretical defects was justified to some degree.

Molodensky's theory gathered isolated parts of geodesy, for example triangulation and leveling, established their connection, freed geodesy from a principal approximate character of its solutions. A more enlightened view was necessary in the modern rapid development of new measurement techniques. Only Molodensky, for the first time, put in front of geodesists and solved a really scientific task: to study the geoid approximately, but to precisely study the Earth surface and its external gravitational field in a uniquely defined coordinate system and to study them as precisely as the measurements permit. The previous theory of a regularized geoid had to be adapted to the geodetic needs, but in fact it did not have a direct relation to the geodetic practice. That is why during the course of the development of geodetic measurements there appeared different improvements, additions, generalizations in Stokes' theory, etc. A harmonic combination of geodetic measurements with solutions of geodetic tasks was realized only in Molodensky's theory. It examines and uses these very measurements, for the present as well as for the future.

From such an understanding of Molodensky's theory, the conclusion follows that it cannot become obsolete by a replacement of one solution method by another or by a modernization of processing methods or by new measurement techniques. New types of measurements (GPS, satellite altimetry for leveling of the sea surface) or planned ones (satellite gradient measurements) are well within the scope of the theory, demanding only an improved precision of it. It is always possible to improve the precision of Molodensky's theory. And that would not be a new theory but only its further development.

A new understanding of geodesy permits to examine any concrete task as a component of the general task of geodesy. For example in the local geodetic works with precision of 10E–6 for transferring an engineering project to nature it is necessary to take into account height anomalies, deviations of the vertical and other realities of the general theory. It is also essential that every task can be solved with a precision permissible by the measurements. So, for example, very rare pendulum surveys permitted to interpolate astronomic latitudes and longitudes, but the modern gravimetric surveys allow in many cases to do without measured astronomic coordinates due to their insufficient precision. Another example: it is assumed in Molodensky's works that the gravity potentials, at all origins of heights reckoning in isolated leveling networks, are equal, and to determine them it was proposed to use astronomic–geodetic networks of different continents. The combination of space observations points with the points of isolated leveling nets gives now the possibility to determine the potentials in these initial points of each leveling network and the possibility to ensure their independent control. It is clear that the general Molodensky theory is only supplemented by this.

The value of the theory is displayed in its unity. The quantities which are to be determined form a system, and neglecting this circumstance can destroy the unity of the theory. So, for example, in some countries up to now no one pays attention to the system of normal heights, examining it as one of the possible height systems. The general significant improvement of measuring precision makes western geodesists pay more attention to the normal height system and to use it as the only one consistent with the general theory (Groten 1995, Yurkina 1996). Measurements and methods of solution will of course improve, taking into account more and more refined effects. Both suppositions underlying Molodensky's theory are being somehow taken into account already now. The principle of the simultaneous determination of movements of measurement points and gravity field changes were envisaged by Molodensky in his article already in 1958. The passage to the global study of the changing gravity at the changing Earth surface will bring geodesy (according to Molodensky — kinematical geodesy) close to geodynamics and astronomy, and the basic theory of gravitation will permit in principle to comprehend in a unique theory the sources of other types of energy. It will permit the future theory of geodesy to enter the future theory of geophysics.

To complete the description of Molodensky's role in formation of the contemporary view on the tasks of theoretical geodesy let us once more look at the works of some predecessors. This is quite necessary because there exist some unclear points in this matter.

All the researches of the Earth figure theory till 1931 were based on the previous simplification of the problem: the Earth is taken as a geoid, and measurements are supposed to be made on it. It is very difficult to establish the consequences of such a simplification. Jeffreys 1931, 1932 for the first time took

Green's formula as a starting point. This formula determines the external gravitational potential of the Earth by its values and its normal derivatives at the Earth surface. Molodensky gave a great importance to a rigorous mathematical methodology, always followed it himself and well appreciated Jeffreys' merit in the monograph 1945. But Jeffreys, as everybody, saw the task of geodesy in the determination of the potential at the geoid, and Green's formula was used by him to prove the importance of free air reduction for the reduction of gravity to the geoid. Jeffreys noticed that this view goes back to Stokes himself who specially accentuated this circumstance in a reprint of his work in 1883 (in the last paragraph of section 13). In other places of Stokes' works there were also similar indications. Nevertheless, when Molodensky's monograph, recalling Jeffreys' works, became known in the West there appeared attempts to denote Molodensky's theory as theory of Jeffreys – Molodensky or simply a New theory, with the understanding that in its development many people took part. This mistake soon was recognized and left long time ago.

As it was mentioned the first theory of geoid for a nonregularized (real) Earth was elaborated by Moiseev, Malkin, and Molodensky. An important step in understanding the tasks of geodesy was made by Moiseev. In his article in 1933 he wrote: "... it was proved many times that with some reasonable assumptions the regularized Earth geoid will very little differ from the real Earth geoid. This does not tell anything about how much will be the deviation of the gravity field for the regularized Earth from the field determined by the real Earth, in the space between geoid and Earth surface. But it is necessary to consider this question as real. The fact is that the geodetic practice often requires, not theoretical gravimetric relations at the geoid, but the corresponding relations at *the Earth's physical surface*, for instance vertical deviations."

Thus it is necessary to recognize as valid the attempt to examine the situation close to the real Earth rather than for a regularized Earth. The contents of this article is devoted to an elementary solution of one of the preliminary questions connected with such an attempt. The same idea was put in a German version of Moiseev's publication in 1934.

Even now, after so many years, this statement remains valid. Still, Nikolay Dmitrievich Moiseev did not try to determine the external Earth potential, in spite of the fact that he saw the necessity to solve this problem, at least for the determination of the vertical deviation. What to do with the geoid height, Moiseev most probably did not know yet. The quotation showed also that only the conceptual difficulties made him use again the smooth surface of geoid as the boundary. Malkin and Molodensky in subsequent publications gave other, more complete solutions, which expressed the height of the nonregularized geoid. (Moiseev deduced only an integral equation for this height). The methodology remained the same but Malkin saw the necessity to improve it. In his articles about determination of a nonregularized geoid, he mentioned the need for modernizing the mathematical apparatus up to a stage when it would not be necessary to change the Earth, even at the cost of complicating the calculations (Malkin 1934, 1935). It is possible to say that these works concluded Jeffreys' attempt to determine the geoid using the real Earth's external potential.

Two articles of Migal 1939 are also related with this matter. He deduced the boundary condition for the disturbing potential *T* at point *A* of the Earth's physical surface:

$$\left.\frac{\partial T}{\partial r}+\frac{2T}{R}\right|_A=-g_A+\gamma_0-\frac{2g_nH}{R} \quad ,$$

where g is the mean value of the gravity inside the continent masses along the height of the point A above the geoid. The height N of the non–regularized geoid above the sphere (ellipsoid) is to be found. It is connected with potential T and orthometric height H by the equation

$$\gamma_0 N = T_A + (g_n - \gamma_0)H \quad ,$$

where γ_0 is mean normal gravity at the height H. The author did not really give a solution of the boundary problem for the Earth surface; in fact, he proposed to use the simple Stokes' formula.

For the customary approximate free–air reduction of gravity to the geoid, $2gH/R$, Migal gave a new interpretation of the free–air anomaly in the second article: he proposed the normal gravity reduction to the Earth surface $-2g(H+N)/R$. It permits to represent the gravity anomaly at the Earth surface as $g-[\gamma_0-(2\gamma/R)(H+N)]$ or $g-[\gamma_0-(2\gamma/R)\,H]$. The author correctly saw the value of such an interpretation in the fact that the masses were not moved at all. It is important for gravitational prospecting. This view at the anomalies was retained in Molodensky's theory.

Malkin 1939 made an attempt to determine the external Earth potential in his short Notice 2 to the article "About the geoid figure determination by gravity observations". We give the text of the notice translated from French almost completely.

"Let us consider the determination of the Earth's physical surface without reductions and condensations. We can suppose that the observation point height above sea level (independently of the true knowledge of the geoid) is known. Then the error in geoid determination causes a similar error in the Earth physical surface. Let n be the normal to that surface S, g_n and δg_n projections of gravity and of its anomaly onto this direction n and onto the plumb–line direction ν. It is possible to write the integral equation

$$\zeta = \frac{\cos(n,\nu)}{2\pi g_n}\left\{\int_s \frac{\delta g_n}{r}\,\mathrm{d}S + \int_s \zeta\left[\frac{g_n}{\cos(n,\nu)}\cdot\frac{\partial \frac{1}{r}}{\partial n} + \frac{\partial g_n}{\partial n}\frac{\cos(n,\nu)}{r}\right]\mathrm{d}S\right\} \quad , \qquad (14)$$

where all values are related to the Earth's physical surface, and therefore there is no need to reduce the gravity to sea level ...".

In equation (14) the author preferred $g_n = g\cos(n,\gamma)$ to the directly measured g. In the next work in 1939, published in 1944, the equation was rewritten in the more convenient form

$$\zeta = \frac{1}{2\pi g}\left\{\int_s \frac{\delta g}{r}\,\mathrm{d}S + \int_s \zeta\left[g\cdot\frac{\partial \frac{1}{r}}{\partial n} + \frac{\partial g_n}{\partial n}\cdot\frac{\cos(n,\nu)}{r}\right]\mathrm{d}S\right\}$$

and it was indicated that it was derived from Green's formula for the disturbing potential

$$T = \frac{1}{2\pi}\int_s \left[T \cdot \frac{\partial \frac{1}{r}}{\partial n} - \frac{\partial T}{\partial n} \cdot \frac{1}{r} \right] \mathrm{d}S \quad .$$

This explanation permits to state that in the deduction of the integral equation a mistake was made in calculating $\partial T/\partial n$. The author denoted the value $\partial g_n/\partial n$ in the article above as "the second derivative of the gravity with respect to the normal to the physical Earth surface". Here we can again see the confusion: it is possible to recognize at once that the author bears in view the second derivative of the gravity *potential*, or even better, the second derivative of the normal gravity potential. In any case there were reasons for the last interpretation in the author's text. But the equation would be incorrect even with such corrections. There is no need to give here the correct derivation of Molodensky's integral equation, because we have already mentioned it. More interest represents another article of Nikolay Romanovich, published together with the previous one (1944 and 1949), "On the calculation of vertical deviations from gravity anomalies in mountains". The disturbing potential is presented as a sum of two terms: Stokes' solution and the potential *T* of a single layer, distributed on the Earth surface in a neighborhood of the point, where the vertical deviation is to be determined. The final formulas for this simplified case were obtained correctly, the measured mixed gravity anomaly is supposed to be equal to the proper anomaly as it is usually made in gravitational prospecting.

Let us mention right here that in the same year of 1944, in a scientific–technical report of TsNIIGAiK, Molodensky not only exposed the main ideas of the 1945 monograph, but in addition gave some variants of transformation of Green's formula, showing interesting ideas about their comparative advantages as well as their links with the analytical continuation of the potential into the masses, touching other questions as well. This part of the report, unfortunately, did not enter into the published works of Molodensky.

The idea of precision of Jeffreys' approach was expressed by Molodensky even earlier. There exists a letter of Molodensky to Eremeev dated January 2, 1943 with the title "Plan of dissertation on the theme: Theoretical bases of geodetic gravimetry". Here is this letter:

„1) Analysis of Stokes' formula

a) Stokes' infinite series
b) Pizzetti's deduction
c) Mixed problem of potential theory
d) Jeffreys' deduction
e) Connection with the integral equations and formulas of Molodensky, Malkin, and Moiseev
f) From the comparison of all the conclusions — why is the condition $g_1 = 0$ inevitable and what does it mean physically?

2) Analysis of models and theory

a) Why does Moiseev's formula not take into account $\partial g/\partial z$, giving less mistakes than Stokes' formula ?
b) To add the influences of the vertical gradient anomaly to Moiseev's deduction and to compare the result with Molodensky's formula
c) To pay attention to Jeffreys' deduction, to try to avoid additional mistakes, to develop it further and to compare it with Molodensky's and Moiseev's formulas.
d) Would it be possible to apply Jeffreys' deduction to Prey's Earth ?

План диссертации на тему:

Теоретические основы Геодезической Гравиметрии.

1) Анализ формулы Стокса.

а) бесконечный ряд Стокса

б) вывод Пизетти

в) внешняя задача теории потенциала

г) вывод Джеффриса

д) связь с интегральными уравнениями — вывод из формул Молоденского, и Малкина, Моисеева

е) по сравнению всех выводов — почему неизбежно условие $g_1=0$ (физически)

2) Модели, анализ ~~[illegible]~~ теории.

а). Почему в формуле Моисеева учет $\frac{\partial g}{\partial z}$ дает меньшие ошибки, чем в формуле Стокса? ~~[illegible]~~

б) Дополнить вывод Моисеева влиянием аномалии вертик. градиента и сравнить результат с форм. Молоденского.

в) Обратить внимание на вывод Джеффриса, попробовать его без дополнительных предположений развить далее и сравнить с ф. Молод. и Моисеева.

г) Нельзя ли вывод Джеффриса применить к задаче Прея?

д) Попробовать решить уравнение Молоденского.

е) В какой мере $\frac{\partial g}{\partial z}$ может сказаться на выводах типа Хирвонена или даже на выводе [illegible] по Клеро?

ж) Почему влияние топографии и $\frac{\partial g}{\partial z}$ на уклон. отвеса входит с разными знаками? Не связано ли это явление с „[illegible]" верш., не являются ли поэтому отчасти результатом преобразования аномалии вертик. градиента?

3) Если ур-ие Молоденского решить не удастся, доказать сходимость процесса последоват. приближений.

4) Правильно-ли не предположить, что вопрос о редукции силы тяжести ~~допол-бывшей~~ к уровню моря может и не решаться строго в общем виде, (если напр. удастся решить ур. ур. Молоденского, да и без этого — напр. топографические поправки). Выяснить соотношение между „топогр. поправкой" и „Geländereduktion".

3) Выводы

а) Можно-ли пользоваться форм. Вейра, а если нет какие поправки в нее внести, поскольку уже идет почти о выводе отличий?

б) Можно-ли пользоваться форм. Стокса при исследовании общей фигуры Земли, и нужно-ли в нее вводить поправки?

в) Правильный подход при исследовании местной фигуры Земли.

Составлено для беседы с Молоденским. Если не придется лично этими вопросами заниматься — поговорить о них не забыть и разобрать в этих деталях сам.

2-I-1943г.

e) To try to solve Molodensky's equation.

f) How $\partial g/\partial z$ can influence Hirvonen type conclusions or even the flattening determination according to Clairaut?

g) Why do topography and $\partial g/\partial z$ influence the vertical deviation with different signs? Is not this phenomenon connected with isostasy — more exactly, is isostasy not partly a result of neglecting the vertical gradient anomaly ?

h) If it is impossible rigorously to solve Molodensky's equation — to prove the convergence of successive approximations.

i) Is the supposition correct that the question on the gravity reduction to sea level need not be solved rigorously in a general form (if, for example, would it be possible to solve Molodensky's equation by the simple use of the topographic correction?)? To find out the connections between "topographic correction" and "Geländereduction".

3) Conclusions:

a) Is it possible to use Clairaut's formula and if not, which corrections do we need to add to it in order to determine the flattening only?

b) Is it possible to use Stokes' formula in investigating the general geoid figure and is it necessary to add some corrections to it?

c) The correct approach in research of the local geoid figure. "

Then an addition comes: "Composed by Mikhail for Volodya. If it is not possible to settle all these questions together — try not to forget and do the best you can".

Probably Molodensky informed Eremeev about the primary plan of his own dissertation: the monograph of 1945. Molodensky proposed Eremeev to study a number of questions which attracted his own interest. Nearly all the questions found their solution in the monograph of 1945 and in the forthcoming works. The necessity to develop and make precise the approach of Jeffreys (item 2c) was clear to Molodensky in the beginning of 1943 and most probably even earlier.

All written here about the works of predecessors shows that it is not possible to consider the idea of Earth surface research completely pertaining to Molodensky. The proposals of Jeffreys, Moiseev and Malkin are not original either. Stokes already in 1849 wrote the following: "We do not, however, want the potential at any point of the interior, and in fact it cannot be found without making some hypothesis as to the distribution of matter within the earth". (It is the text of 1849). But it appears that Stokes' important remarks were completely forgotten. The outstanding geodesists of the past understood that the Earth figure and its external gravitational field determination was not possible by means of the geoid and orthometrical heights. This was mentioned by Helmert 1884, p. 502 and by Pizzetti 1906 –1925, p. 206. In the last century only Jeffreys, Mikhailov, Moiseev, Malkin, and Migal to certain degree understood the necessity to modernize Stokes' theory of geoid determination and made attempts of at least its partial solution. But none of them did not even come close to a solution of the problem, not to speak of creating a unified theory for the whole modern geodesy. This great task in the full generality was elaborated almost quite exclusively by the works of Mikhail Sergeevich Molodensky.

Nowadays Molodensky's theory is recognized. But the recognition came with many difficulties, giving many sad minutes to Molodensky. Chebotarev, who by that time was one of the leading MIIGAiK's professors, after Molodensky's report said: "The matter disappeared, only the formulas are left", and at one of the geodesy section meetings he also said: "The baby was thrown out of the tub together with the water" (Chebotarev meant the rejection of the geoid as the main surface representing Earth). Molodensky answered to this statement quietly, without pathos: "The continents remained at their places", "The work was made on the base of Newton's law". Just after the defense of the doctoral dissertation Molodensky's work was sent from MIIGAiK to the Academy of Sciences for examination. The work was highly appreciated by the experts and Molodensky's report at the Presidency of the Academy contributed to his election as a corresponding member. For the first time the advantages of the normal heights over the orthometric heights were officially approved. At a special meeting of the geodetic section of TsNIIGAiK's Scientific Council with the support of Danilov, Larin and Chebotarev, there was approved Eremeev's work published in 1951 about the practical calculation of the normal heights and comparison of them with the orthometric and dynamics heights. The normal heights entered the geodetic practice of the USSR. The normal heights system was officially introduced in France (Kasser, 1984) and in Germany (Grote, Denker, Torge, 1995). It was indicated in the last article that the introduction of normal heights was to be expected in Europe. (The normal heights were introduced also in China (Chen 1994)). So, in discussing geoid determination, very often one means the application of Stokes' theory only as the first approximation to Molodensky's theory. Only the reductions necessary in Molodensky's theory are being taken into account. The topographic–isostatic reductions are being used as an interpolation method. Only in Austria, as far as it is known to the authors, a special map of distribution of density in the Earth's crust for calculating the geoid figure and orthometric heights exists, because neglecting such corrections leads to errors in geoid heights and vertical deviations up to 20 cm and 3'' (Suenkel, 1986).

Molodensky's works left a permanent track not only in geodesy. He devoted the second part of his life to Earth physics, without leaving geodesy, studying elastic oscillations, rotation and structure of the Earth as expressed by its free oscillations. The necessity of such investigations is clearly a task of theoretical geodynamics as well as a means for determination of data required by other disciplines occupied with global research of Earth phenomena. The Americans justly link these questions with geodesy.

The research of movements and deformations of the Earth must be referred to an immovable system of coordinates. Otherwise coordinate changes may be erroneously attributed to some real forces. The quasars, distant from the solar system not less than 700 millions of light years, are the most stationary objects on the celestial sphere. On their base the celestial coordinate system was established with a precision up to 0'',002. The other coordinate system, the terrestrial system, is determined by the points at the Earth surface from laser tracking of satellites with an error of about 5 cm. The elements of mutual spatial orientation are necessary for connection of the celestial and the terrestrial coordinate systems. The Earth's rotational axis plays a great role in this context. It is necessary to know the proper Earth movement relative to the rotational axis (polar motion) and the angular speed of rotation. The specific complexity of this task is connected with the fact that, having only the measurements at the Earth surface, one cannot separate the polar motion from the Earth surface deformations and orientation elements (angles of procession and nutation). This is why it is not possible to check the nutation and precession theories by terrestrial observations. To calculate the Earth's spatial movement, one needs data about its internal structure and a theory of Earth movement in space, and such a theory must reflect properties of the real Earth. In modern calculations one uses the best data available for the distribution of density inside the Earth; they are obtained from the results of free oscillations and seismic wave distributions. They take into account the form of the boundary between the Earth core and mantle (the relief of this boundary according to some evaluations goes up to 10 km). They also take into account deviations of the Earth's rheology from Hooke's law and elasticity coefficients of Lame, the influence of sea tides and other effects. The use of space methods for surveying the Earth's relief from outer space (or the use of an air survey) could help determining the polar motion relative to the Earth relief, similar to methods proposed by Ziman, Nepoklonov and Rodionov (1970) for the planets. This could permit to exclude the polar displacements relative to the relief, leaving the astronomic observations to be used immediately for determination of the Earth's spatial orientation. But such an approach has not been adopted so far, although its theory is being developed (Urmaev and Frolov 1999).

Of course, the precision of the theory must correspond to the observational precision and even be better. Molodensky's works represent an important stage in the elaboration of such a theory in 1953–1989.

The forces which act considerably longer then the time of relaxation of strains, can be regarded and examined as hydrostatic effects, considering the Earth for this purpose as liquid. If the change of the density with the depth is known, it is possible to calculate the flattening of the layers of constant density and the corresponding strains. These solutions (Clairaut – Radau theory) are considered as the initial conditions for Molodensky's theory. Further objects of research are the small elastic oscillations of a spherically symmetric, rotating Earth with a liquid core in a coordinate system having nutational movement (the precessional movements for a liquid and a solid Earth are identical).

The principle of solution of this problem by an expansion into series of solid spherical harmonics was described by Molodensky in 1953. The elementary components of oscillation are expressed through the radial displacement, volume expansion and change of attraction potential, which are proportional to Legendre's associated functions. Molodensky rejected the variational method developed by Jeffreys and Vicente 1957 and in 1961 described a new way of constructing a theory of Earth nutation and diurnal Earth tides. Molodensky and Kramer managed to obtain good agreement with the observational data on the real Earth. For example, they obtained 433 days (model 1) and 436 days (model 2) for Chandler's period of polar motion. This theoretical model of Chandler's motion remains the most precise up to now. For comparison let us mention the following observational data from Pulkovo: 434, 3 days (Kostina, Sakharov, 1979). Evaluating ILS/IPMS data for 1900–1973 by the method of maximum entropy, Currie 1974 found 432,95 mean sun days. Molodensky's solution 1961 was described by Moritz (1980–1982) in the Reports of the Department of Geodetic Science and Surveying of Ohio State University, Columbus. This description entered into Moritz' and Mueller's book 1987. Molodensky's nutation calculation was recommended by the International Astronomic Union at the General Assembly in Montreal in August 1979. But the density distribution inside the Earth adopted by Molodensky does not exactly correspond to the actual data, so that corrections were needed. As Smith 1981 stressed, the development of Molodensky's 1961 method in particular in the works of Sasao, Okubo, Saito 1980 and Wahr 1981 consisted in general in increasing the precision of the Earth models. Molodensky's results recalculated with improved data on the Earth's structure lead to a good agreement with the results of later more complete theories. Such agreement was mentioned in particular by Wahr 1981. The difference between Molodensky's and Wahr's results makes about 10% of the total correction for non–rigidity or 0, 002" at 6 months or 18, 6 years. Wahr explains these differences in principle by the imperfect Earth model used by Molodensky and to a lesser degree by the theory itself. These disagreements are somewhat larger than the influences of ocean dynamic for nutation and probably larger than the core and mantle coupling.

Molodensky's nutation calculation theory is appreciated in Moritz' and Mueller's 1987 book as a standard for the all following works, and his numerical results are called excellent even by modern standards.

In 1961 Molodensky calculated also the period of the nearly–diurnal nutation of the mantle, caused by the liquid core.

In many subsequent articles Molodensky continued to modernize the theory. Coriolis forces were included into the equations for taking into account the rotation of the coordinate system connected with the Earth (1970). This makes the theory applicable for studying low frequency oscillations, where Coriolis forces have a basic influence. The solution was constructed without the previous separation of spheroidal and toroidal oscillations (1972). His unified method involved oscillation of all kinds and all frequencies, including forced and free nutation of the rotational axis and changes of the Earth's rotational velocity. The solution led to a system of ordinary differential equations for the elementary components of the oscillation for each harmonic (1974, 1976, 1977). Five constants enter the solution of each system of equations. They characterize some effects, on the surface, of the elastic oscillations of the whole Earth and are basically very similar to the Love numbers. The traditional use of the Love numbers is connected with a hypothetical assumption mentioned by Love himself. The author proposed to maintain the previous name, Love numbers, for those new five constants.

The modernized theory of Earth oscillations was described by the author in his last monograph 1989, which also contains new results. In particular, Molodensky used perturbation theory to take into account the deviations in the Earth structure from the spherically symmetric model. The results were extended to the case of a viscous Earth core. The Earth structure determination on base of the free oscillation frequencies leads to inverse problems with unstable solutions. Molodensky proposed the method of eliminating spurious density fluctuations. His last ideas did not yet find their use. Molodensky's works are again ahead of the time, they form a base for a more precise theory necessary for closing the precision gap between modern techniques and new theory, and that is why the needs of geophysics logically call for the use of the last ideas of Molodensky.

Researches on the theory of rotation of a deforming Earth, its oscillations, the theory of its figure and gravitational field give invaluable contributions to the development of the fundamental Earth sciences, and are necessary for the full use of new techniques. Besides the astronomic–geodetic theory of Molodensky, his theory of elastic oscillations will find a use in other fields of knowledge — geotectonics and in the theory of the major planets.

Devotion to the science combined in Mikhail Sergeevich Molodensky with the delicateness in his relation with people, his openness, and his simplicity in life. He was a good family man, together with Alexandra Mikhailovna they raised two daughters and two sons who occupy dignified places in life. The people who worked together with Mikhail Sergeevich are thankful to their destiny. He always made everyone admire his daring ideas and the elegance, rigor and completeness of his researches.

The scientific world community highly appreciates Mikhail Sergeevich's works in the Earth sciences. The President of the International Union of Geodesy and Geophysics, Helmut Moritz, asked the Interministerial Geophysical Council at the Presidency of the Russian Academy of Sciences to lay down at Molodensky's tomb a wreath with the inscription "To the great scientist M. S. Molodensky in gratitude the International Union of Geodesy and Geophysics".

In conclusion we express our thanks to Sergey Vasilievich Kusakin, a specialist of the region and teacher at the school of Epiphan where M. S. Molodensky studied, for the information about the teachers of M. S. Molodensky and about Epiphan.

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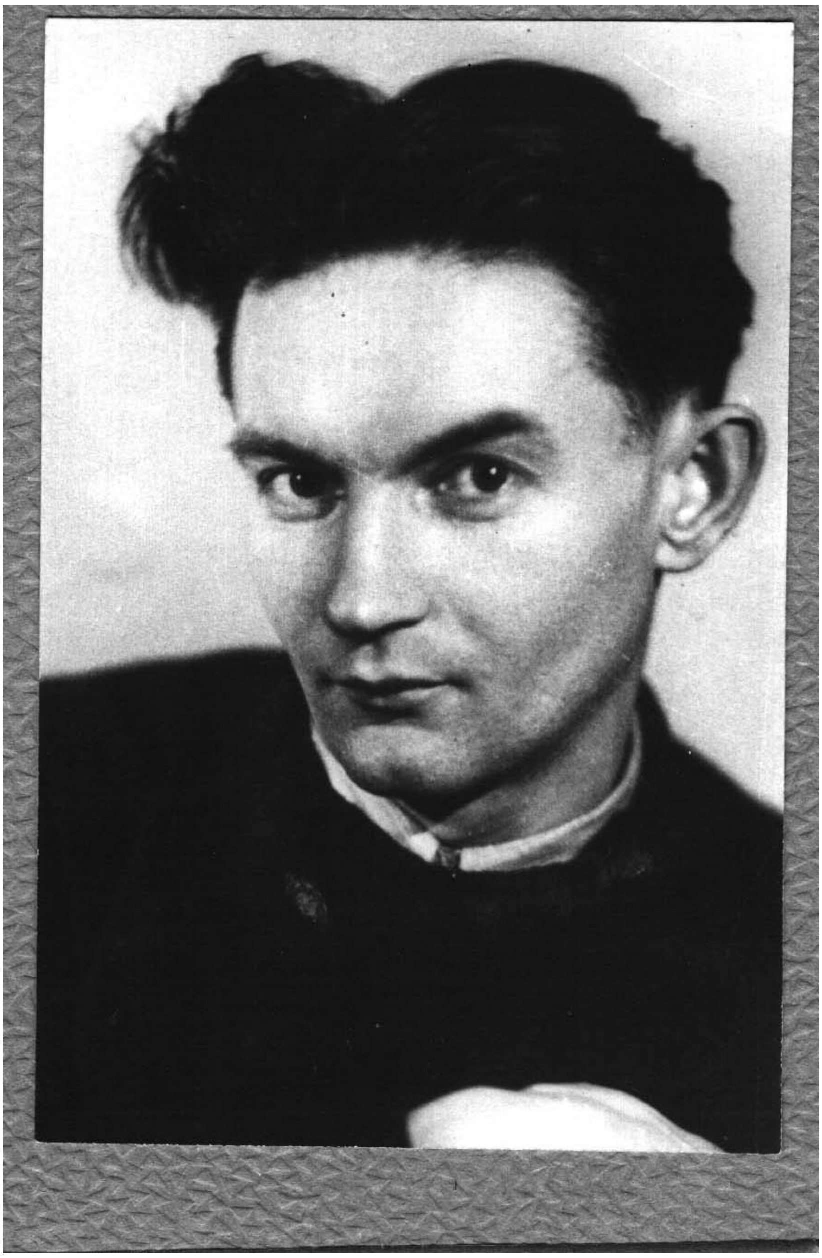

IN MEMORIAM OF A SCIENTIST

M. S. Molodensky and his way in science

By M. Heifets

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In the second part of the 1920ies at the height of the NEP (New Economic Policy) launched in the country, there started a preparation, hardly noticeable but steady, for the fulfillment of Stalin's five–year plans. One of the important stages of the new policy was to train specialists of a worker–peasant origin, who had experienced a political upbringing by party functionaries of high ranks. The first steps taken by the political nomenclatura for this purpose was to set up some privileged party groups, so–called "party–thousands", at all higher schools as it was believed that a party–membership card could guarantee mastering any branch of knowledge and any profession quickly and without problems. The "party–thousand" was charged to control a "trained–staff policy" at higher schools in the country, which meant to limit the access to higher education for intelligentsia, employees, and persons of free professions, and not to allow entering higher schools at all to children of merchants, noblemen, clergymen and former officers of the tsarist army. The "party–thousand" was assigned to discover students who concealed their social origin (this was a mass practice among students despite the threat of being thrown out from the institution they studied at). In its turn this lead to a practice of informing on each other which not only was not disapproved but, on the contrary, was even encouraged. Thus all the measures were taken to make entering higher schools much easier for factory workers and village peasants who, as a rule, were semi–literate.

As a result of this system of education with "rabfaks" (workers' schools) and privileged courses, the higher schools were filled with people whose knowledge and intellect were rather poor. For the sake of promotion of these people, the former education system, well–developed and proved, had been abolished and substituted by an artificially created system, the so–called "laboratory–brigade method", which totally ruined the higher education system for four years of the "method's" existence. It was stopped only in 1932 by N. I. Bukharin, who headed the USSR TsIK (Central Executive Council) Committee on reformation (actually, on restoration of the higher school system as is had formerly existed). In order to characterize, in short, the laboratory–brigade method it is sufficient to say that, in fact, there were no general lectures delivered for the whole course as all the course students were divided into separate groups ("brigades") where they had their lessons. One student of a brigade was assigned to prepare, on behalf of the whole brigade, an appropriate part of the course that a professor would have delivered on a certain discipline. Taking into account the dramatic lack of textbooks at that time it is easy to understand that such lessons were rather a formality; it is obvious that the brigade students' reports did not reflect any new developments in science and technology that could have been introduced by the professor. The functions of a professor were now extremely limited – he had to supervise lessons and answer questions addressed to a student–lecturer if the latter was incapable of doing it himself. Professors had no possibility of making use of their erudition — they were cut off from the majority of the students. Grades were given by the students themselves by means of a mere voting, and the student's success in studies was determined by the word "passed". The students had the right even to dismiss any undesired teacher, and this procedure was also executed by a vote.

This right was applied quite often, especially in cases when a teacher, in the opinion of the most zealous "party–thousand" men, did not seem good enough at political education and happened to say something incorrect from an ideological point of view, even if it concerned a subject far distant from politics. The students themselves worked out their curricula and arranged their educational process. They used to elect a pro–rector who was responsible for students' affairs and was freed from lessons for a one–year term to control the observance of the students' rights. In those days all university degrees and titles were abolished. Some professors who aroused suspicion with regard to their origin, regardless of their scientific merits, were treated without pity: they were deprived of their food–cards and forced to buy food–stuffs at free market prices much higher than the prices fixed for the privileged ...

N. I. Bukharin was the first to take the risk of taking upon himself the responsibility for putting to an end the existence of this "theatre of absurdity", and one cannot help admiring his courage.

Fortunately for Russia, even at that terrible time there remained many pre–revolution specialists whose greatest concern was to benefit their country. It was they who, having responded to the country's call, had provided a rapid growth of industrial potential in many branches of national economy and science. This also was the case with the development of gravimetry and gravimetric surveys. Many prominent scientists, A. D. Arkhanghelsky, G. G. Gamburtsev, A. A. Mikhailov, B. V. Numerov, O. Yu. Schmidt and others were engaged in the solution of scientific and technological problems and did their best to advance Soviet gravimetry to the first place in the world. By the early thirties the Soviet Union had measured twice as many gravimetric points as all the other countries of the world together.

Mikhail Sergeevich Molodensky was born on the 15th of June,1909 in the town of Epiphan of the Tula province, in the family of a priest. When he was 14, he moved to Tula, where he started living with the family of his uncle, a medical doctor, and studied at a railway school. Having finished the school, he was not able to enter a university institute because of his young age, for, according to age qualifications existing at that time, the age for leaving school was raised to 18 years. So he had to work as an accountant in the forestry of Yasnaya Polyana, not far from Tula, for about a year. It was only the beginning of his long struggle against major difficulties that he was obliged to overcome in order to realize his striving for scientific knowledge, especially for astronomy which had attracted him since his childhood.

The most serious obstacle in his life was his social origin. Nevertheless, in 1927 he managed to enter the astronomy department of Moscow University. The faculty dean met him with obvious displeasure, and he had to overcome obstacles in his studies for a long time. M. Molodensky was neither given a scholarship nor a room in the student dormitories. However, all the attempts to remove M. Molodensky from the university were unsuccessful. Molodensky's authority among the students and his reputation had grown greatly, and when he was a fourth–year student he was elected a monitor of his academic group and a student–union leader.

In 1930 there was an attempt to remove astronomy from the curriculum for it was considered to be less essential. The students specializing in astronomy were offered to change to another discipline, called “geodesy and gravimetry”. But the group headed by M. Molodensky, where V. F. Eremeev, L. Petrov, M. Zverev and others, rejected this suggestion and decided to combine their studies in astronomy with those in geodesy and gravimetry. The leading teachers of the group were Prof. F. Krasovsky, M. Solovyev (in geodesy), A. Mikhailov, L. Sorokin (in gravimetry and the theory of the Earth), and S. N. Blazhko (in spherical and practical astronomy). Eventually the teachers’ intellect and the monitor’s enthusiasm had won and the students together with their main teachers overcame the obstacles on their way to high education, at the risk of their future positions under these difficult circumstances.

After graduating from the University and getting their diplomas in astronomy, all of the students later became distinguished specialists in gravimetry, geophysics and astronomical geodesy. As to M. Molodensky, he was given his diploma within one year, in 1932, when he published his first scientific work “On latitude calculation by observation of Pevtsov pairs” in the Soviet “Astronomical Journal”.

Molodensky’s first steps of his activity in geodetic researches and surveys dated back to 1929 when he started surveys headed by D. Sherman at a test area of the Institute of General Geodetic Surveys (IOGR). Their joint work went on for two years. Before it, M. Molodensky had got a proposal from the director of the Astronomy–Geodesy Research Institute (AGNII) at the State University in Moscow (MGU) to have a permanent job there after graduating from the University. Some time later this institute was combined with the Astrophysical Institute of MGU, and the newly formed institute was called “State Astronomic Institute named after P. Shternberg” (GAISh). There M. Molodensky was appointed. There he first took part in gravimetric measurements made by a joint expedition of MGU and MGI (now MIIGAiK) headed by N. Parijsky and K. Shistovsky, in the Ural, where M. Molodensky gained much experience as a gravimetrist. At GAISh M. Molodensky also was engaged in some astronomic research works in the Kuchino observatory till 1935. Gradually his activity tended to shift to geodetic gravimetry and to the theory of pendulums, which resulted in his leaving GAISh.

By that time a famous decree of the “Soviet of Labor and Defense” about the establishment of a general gravimetric survey over the whole country had been issued. The realization of this decree provided a great increase in the development of gravimetric surveying works all over the country.

M. Molodensky, being keen on gravimetry, eagerly participated in these surveys, and he started work at TsNIIGAiK which at that time was in charge of providing scientific and methodological guidance for all gravimetric measurements over the country. An exclusive right to carry out gravimetric surveys was given to the Main Department for Geodesy and Cartography (GUGK) under the Narkomtyazhprom (People’s Committee on Heavy Industry). Other departments and surveying agencies could take part in the work under the control of GUGK and at its expense. In fact, gravimetric surveys were carried out by IOGR, which finally was transformed into Moscow Aerogeodetic Enterprise (MAGP).

In 1933, M. Molodensky headed an expedition to the Crimea to perform gravimetric surveys according to the above Decree. The experience M. Molodensky had gained in the expedition was so rich and fruitful that he, a young specialist in gravimetry, was included into a special commission on the elaboration of the first instruction for a general gravimetric survey of the USSR.

In 1934 the name of Molodensky became known abroad, as he made a report at the 7^{th} Baltic Geodetic Commission Conference in Moscow on an urgent problem that had been, for many years, of great interest for gravimetrists, that is, the co–swinging influence on double pendulums. Before, it had been practically impossible to determine the accuracy of the numerous solutions in existence, as all of them had been approximate. M. Molodensky was the first to find a rigorous solution. His report was listened to by many distinguished scientists from Baltic countries, Denmark, Finland, Germany, Poland, Sweden, the USSR, as well as by members of the International Association of Geodesy. Molodensky's report on the conception of the astronomic–gravimetric leveling was published in 1937 in transactions of the 9^{th} session of the Baltic Geodetic Commission (Helsinki 21.–25.7.1936).

Despite his brilliant capabilities and creative activity characterizing M. Molodensky as a scientist, his personal life and his advance in science were not easy at all. At TsNIIGAiK he seemed to have got a chance of working in his profession, but there again he faced a hostility on the part of the administration of the institute. The explanation came from his social origin and from the fact that was characterized, on the one hand, by his full absorption in science, high nobleness and honesty, and, on the other hand, by his disgust for any lie and hypocrisy and his incapability of ingratiating himself to anybody. In other words, his behavior did not conform to the then–existing standards, and therefore, he could not benefit of any rapid progress in his career. Fortunately, the situation radically changed when in 1939 a new director, A. Tatevyan, was appointed. A. Tatevyan, being a talented scientist and an honest man, quickly recognized M. Molodensky's personality and later offered him his utmost assistance and support, and protected him from those who considered a "public activity" more important than a scientific one.

The contribution by M. Molodensky to the development of Soviet geodesy and gravimetry is great, both in theoretic researches and in gravimetric surveys, especially in developing and applying advanced surveying methods combined with designing precise gravimetric instruments. Such instruments were of particular importance. They required a base for production and designing, involving hundreds of highly trained people. In fact, in the early thirties there were only gravity measuring instruments of foreign production available in the country, and most of them were out of date. This is why an immediate task facing Soviet geodesy was to manufacture appropriate instruments. The first small batch of pendulum instruments was produced by the plant "Aerogeopribor" in 1935. They were a slightly improved copy of German "Bamberg" instruments. Several attempts to design original instruments at other enterprises had failed, and only in 1938 a series of good light pendulum instruments designed by L. Sorokin was manufactured at GAISh.

In April 1943 M. Molodensky was appointed chief of the gravimetric laboratory at TsNIIGAiK. He continued to occupy the post till July 1956 when he was appointed, against his will, director of GEOFIAN (Geophysical Institute of the Academy of Sciences). Molodensky was responsible for realization of the technical policy pursued by GUGK in gravimetric surveys. It is obvious that, being interested in the progress of surveys, GUGK had to face various problems, including improving the methods of gravimetric measurements. GUGK's activity at that time contributed much to the development of gravimetry: it had close contacts with many research institutes in the country, such as the USSR AN Seismological Institute (now the AN — Academy of Science — Institute of the Physics of the Earth's (IFZ), GAISh, LAI (Leningrad Astronomical Institute), later ITA (Institute of Theoretical Astronomy), and many other geological prospecting institutes and organizations; many gravimetric conferences and seminars were held in Moscow and other cities of the country, stimulating a creative search for new ideas in gravimetry.

But the leading part in the development of Soviet gravimetry belonged to TsNIIGAiK, actually, to M. Molodensky who had managed to find some ways of eliminating a serious handicap in gravimetry caused by the lack of its instrumental base. His first step in this direction was to complete the laboratory staff with qualified gravimetrists. Very soon there came V. Goroshko, N. Grushinsky, N. Linnitsky, A. Lozinskaya, M. Makover, G. Razdymakha, G. Rudakovsky who were to develop or make copies of new gravimetric instruments of different types: pendulum instruments, elastic pendulums, gas and quartz gravimeters. Of all the attempts made by these gravimetrists, the most successful were elastic pendulums designed by G. Rudakovsky. Since 1937 and during the war were used for surveys in regions difficult to access (Caracumy desert, polar areas of Yakutia, the Caucasus, the Pamir etc.), being comparable to usual pendulums in their precision. The most important development of TsNIIGAiK that had changed the whole process of gravimetric surveys in the USSR, was a new static gravimeter designed and manufactured by M. Molodensky with the assistance of a team consisting of A. Lozinskaya, N. Grushinsky and a mechanic V. Gushchin in 1938 – 1940. Series production of the gravimeters started at the height of the war. The first production batch of gravimeters of GKM index consisted of about 100 units, and was manufactured under the guidance of N. Sazhina and L. Kalisheva. The next model, a more improved one, was developed by a research team headed by A. Lozinskaya. Then there appeared two modifications of these instruments: one was designed for sea–bed measurements, the other one, for test aerogravimetric determinations. The great success of the Soviet instruments was recognized by the State Prize awarded to their designers, M. Molodensky, A. Lozinskaya, L. Kalisheva, N. Sazhina, L. Sorokin, V. Fedynsky, and to a number of other scientists, designers and engineers involved in the instrument production.

The emergence of precise and cheap gravimeters produced in the country after World War II lead to an intensive development of gravimetric–prospecting surveys, and soon gravimetric measurements numbered one million points per year, hundreds of times exceeding the rate of development of the general pendulum survey. The Soviet efforts in solving global geodetic problems, began with reconnaissance surveys, and, since 1937 created the state gravity system where general survey points had been divided into four classes already at that time.

M. Molodensky with other scientists was very much concerned about the quality of rapidly–accumulated gravimetric points, and in 1937 he headed a commission aimed at solving two problems: to replace obsolete pendulum gravimeters, made in Russia and abroad, and to classify and catalogue all pendulum–measured gravimetric points over the country, and, if necessary, to re–process initial data. To cope with the tasks requiring much laboratory work, a special bureau headed by N. Kiselev, and afterwards by I. Kazansky, had been set up where, under the general guidance of M. Molodensky and M. Zverev, many prominent gravimetrists took part in the work. By this very bureau there were first developed and tested new methods of data processing and accuracy estimation for pendulum measurements, means of leveling calculations, techniques of compiling gravimetric catalogues, rational methods of processing astronomic observations and of determining barometric heights in gravimetric surveys.

A thorough analysis of pendulum data that had accumulated since the second part of 19th century, revealed which instruments needed to be rejected because they could not meet the accuracy requirements stipulated by the Instruction of 1937. The process of eliminating such instruments of course slowed down the speed of surveys, and the State plan fulfillment was on the edge of failing, but on the other hand, it improved greatly the quality of surveying and brought a lot of benefits. Making such a decision in the times of the thirties in our country called for exceptional courage and strict adherence to his principles on the part of M. Molodensky, especially if we take into account the attitude of the Institute administration towards him.

This new approach to the general survey made it possible to refine its "zero–point", and an initial value for this point was chosen very carefully and thoroughly, fixing for many years ahead the value determined by N. Parijsky in 1935. An accuracy for the points of all classes was not yet specified, though requirements on the methods of their determination were strict and high. It was only in 1944 when error tolerances for the point measurements first were stated in the Instruction of the same year.

Joint efforts of M. Molodensky, M. Zverev, N. Parijsky and Yu. Boulanger to estimate accuracies of pendulum measurements brought a basic and simple solution of this rather complicated problem. A correlation between a priori and actual errors was made for about 10000 points, and their values fully coincided. The error estimation showed that the precision of field measurements had a tendency of constant improvement although measurements moved on to places that were difficult to access. This tendency was very important, since gravimeter measurements carried out during the time after the war were more precise than pendulum observations, and the fate of pendulum surveys hung on a thread, as there were many serious doubts about it at that time.

Meanwhile, the fame of M. Molodensky as a scientist grew rapidly. In 1943 he was invited to deliver lectures in gravimetry and the theory of the Earth's figure at the Moscow Institute of Engineers for Geodesy, Airsurvey and Cartography (MIIGAiK) where he worked till 1947. But teaching, in general, did not interest him very much, and he considered this job as a mere duty. In 1946, with great enthusiasm, he started work in the gravimetric department of the Geophysical Institute at the USSR Academy of Sciences (GEOFIAN), as a department chief, combining two jobs.

In fact, he did not give up his main job at TsNIIGAiK despite hard conditions of working and living. At that time the Institute was housed in several buildings situated in different parts of Moscow. M. Molodensky with his family of six members had to live in the outskirts of Moscow, in a former village, in one room of an old wooden house, far from TsNIIGAiK, MIIGAiK and GEOFIAN.

Nevertheless, his activity was very fruitful. Each of the researches carried out by him was characterized not only by depth of theoretical analysis, but also by its practical significance and usefulness (more will be said about this below). The efficiency of his researches always was accompanied by an efficiency of organizing the work proposed by him (in fact, the two abilities do not very often occur together). The realization of his proposals brought great benefit to the progress of Soviet gravimetry, which has been generally acknowledged throughout the world.

The rapid introduction of the gravimeters developed by M. Molodensky made it possible for him to implement, in the course of surveying, some important ideas in theoretical geodesy which were of great significance for the development of the Earth sciences in general and for topographic–geodetic study of the country's territory in particular.

Here we should say some words about a concept that was developed by M. Molodensky during war time (but starting much earlier) concerning the study of the Earth's physical surface without using information on its inner structure. It is impossible to overestimate the significance of this concept for science and national economy: it meant a revolutionary reform in geodesy. For this work M. Molodensky was awarded the USSR State Prize (1946), he got the high degree of a Doctor of Technical Sciences, and was elected a corresponding member of the USSR Academy of Sciences (AN).

This direction of researches by M. Molodensky was a realization of F. Krasovsky's idea of using gravimetric data for processing astronomic–geodetic measurements. Molodensky elaborated a possibility of using gravimetric survey data for a detailed interpolation of plumb–line deflection between astronomic points of astronomic–geodetic networks. Such an interpolation is achieved with a minimum amount of additional gravimetric measurements, provided a general survey of the region has already been executed. In this case, the gravimetric survey includes densification of gravimetric points around astronomic points with plumb–line deflections determined for these astronomic points. The result of this work permitted to integrate some isolated sections of the astronomic–geodetic network into the main system of coordinates, and this, in its turn, made it possible to map vast areas in the Far East, which had never been surveyed before. The process of geodetic coordinate determination suggested by M. Molodensky was incorporated into a method of astronomic–gravimetric leveling of geoid heights that had been developed also by him. Such leveling allowed a mapping of the heights necessary for astronomic–geodetic network data to be processed by a method of projection just as it was conceived by F. Krasovsky. Increasing amounts of data obtained from detailed and precise reconnaissance surveys requiring just a small additional processing, contributed much to the astronomic–gravimetric leveling of the country.

Before the sixties, point densification was made for all the points of interest to geodesists, within a radius of 60 km from an astro–point, irrespective of location of geophysical points. Where both data from 1:200 000–scale maps and data of geodetic densification surveys had proven to be of the same practical value, plumb–line deflections were measured on the basis of combined data acquired by both geodesists and geophysists alike. Only in cases of close proximity to an astro–point where maximum accuracy in gravity gradient determination is required, geodesists went on measuring separately.

Thus, owing mainly to theoretical and instrument–design achievements by M. Molodensky, gravimetric objectives facing geodetic services essentially changed in the post–war years. Gradually the general survey came to an end, and by 1957 it had been completed. The efforts of surveying enterprises within the framework of GUGK were focused on astronomic–gravimetric leveling. The efforts of TsNIIGAiK concentrated on solving complex problems associated with the theory of the Earth's figure, on space explorations and defense problems, and on the development of triangulation methods for large territories. Many prominent scientists were involved into the work by M. Molodensky. At first, they were B. Dubovsky, V. Eremeev, A. Lozinskaya, and later, L. Pellinen, M. Yurkina, O. Ostach, and G. Demianov.

However, M. Molodensky's activity was by no means restricted to the areas just mentioned. He was known as a brilliant generator of various scientific and engineering ideas and initiatives, but he was modest enough not to demonstrate this openly, preferring to stay out of the limelight. It is hardly known to anyone that he was the first to put forward suggestions of calibrating gravimeters by tilting; of devising a highly–effective method of simultaneous observations of four pendulums; of introducing the notion of "control number", splendid in its simplicity; of a simple but rigorous processing method for measurements by elastic pendulums; as well as of a rapid but precise method of astronomical coordinate determination that was immediately put into practice during the war. These methods and procedures, registered nowhere under his name, brought benefits and high economic efficiency (four–pendulum observations reduced by a factor of two the time of measurements at a point; gravimeter calibration by tilting made field measurements for determination of scale coefficients almost unnecessary), but the author, deserving the right to be rewarded, had never taken an opportunity of it.

Here, we cannot help speaking once more about M. Molodensky's boldness and foresight. In 1948 at TsNIIGAiK there was a suggestion by M. Heifets to develop and bring into practice quartz–metallic pendulums made of three different materials. Gravimetrists and instrument–manufacturers were categorically against the idea. M. Molodensky was the only one who provided full support to the initiative. Even when the first series of 12 pendulums had appeared to be unsuitable, M. Molodensky went on insisting on their further development and use, despite many years of resistance on the part of state manufacturers. Nowadays the whole first–order network of the country has been established using such pendulums, and the network is known to be one of the best in the world.

In 1955 TsNIIGAiK, lacking both appropriate instruments and trained staff, took upon itself the task to perform a gravimetric survey of the world oceans, and two expeditions were sent out to make underwater measurements. In seven years the survey was completed in the first approximation. M. Molodensky did not participate in the survey, but his moral support of the project was of great importance.

M. Molodensky was the initiator of another very difficult project, an aero–gravimetric survey of the Arctic region and the Antarctic continent.

In the last decade of his life M. Molodensky, being struck by a heavy disease, continued his researches in theoretic geophysics. His success in the development of methods to determine the gravitational field and the Earth's figure, as well as in the theory of terrestrial tides was recognized by the government: he was awarded the Lenin Prize.

In geodesy as a science, M. Molodensky is a unique phenomenon, and geodesy is forever connected with his name. It was he who had laid the foundations of its newest and most important directions, such as kinematic and dynamic geodesy, having created an independent discipline, geodynamics, one of the most promising sciences of nowadays, full of new problems, that was formed at the junction of geophysics, geodesy and astronomy. Many of M. Molodensky's followers and disciples whom he helped understand and master new concepts, notions and factual data first introduced by him, are working now in this branch of science. Gradually but very slowly, these notions became known abroad. "The basic problems of geodetic gravimetry" by M. Molodensky (1945) was first published in the German Democratic Republic in 1958. Next year the work was translated into English in the USA. Summaries were published in Poland and Czechoslovakia in 1953; later on, in a much shorter form, it was issued in the USA in 1959 and in Germany in 1961. The book "Methods for study of the external gravitational field and figure of the Earth" was published in English in Jerusalem in 1962. Thus, information on the greatest achievement in theoretical geodesy had been reaching the Western countries only after 15 years, sometimes in a deformed interpretation because of the complexity of the matter. Even now, when M. Molodensky's merits have been recognized in the world, some misunderstanding and silence about his works still occur.

M. Molodensky strove to render the greatest possible assistance to everybody who asked him for help in theoretical and applied problems. Moreover, a considerate attitude towards people always was an unalterable law for him, the same as kindness, good–heartedness and modesty. After he had got a world–wide reputation, high titles and positions, he remained the same as he had been before. He suffered a lot of financial difficulties, but refused to defend a thesis to get a university degree

Отзыв

о курсовом проекте студента Милоша Бурша М.: „Проект астрономо-гравиметрического нивелирования на территории Чехословакии".

Проект производит чрезвычайно хорошее впечатление своей целеустремлённостью, ясностью и чёткостью изложения. В первой части (теоретическое обоснование проекта) изложено всё необходимое и ничего лишнего и с полным пониманием дела. Все мои замечания носят главным образом редакционный характер и сообщены устно автору проекта. Первая часть проекта заслуживает отличной оценки.

О второй части проекта судить не могу.

Молоденский.

26-II-1955.

that could have improved his condition. Eventually, in 1938 he was given the first higher degree without defending his thesis. Even after he had become a corresponding member of the USSR Academy of Sciences and been awarded with the State Prize of the USSR he continued living in a small room of a wooden hut–like house, never demanding anything for himself. At one time he decided to move to the town of Poltava where he was promised a job in a gravimetric observatory, with a flat of his own. Only then O. Schmidt, director of the Institute of the Earth's Physics where M. Molodensky had some additional work, ordered to give him two rooms at his disposal in a flat with some other tenants living there. Several years later this lodging problem was successfully solved at last for M. Molodensky's family.

M. Molodensky's behavior, in fact, was different from others in his non–standard style of life. He rejected many privileges which others could only dream of: he refused to be promoted to an academic title for a long time, and he avoided travelling abroad while others strove to do this. He was extremely modest in his every–day life, being content with very little: a simple meal, clothing, furniture. In the winter of 1932–33, after having graduated from the University, when he worked in a laboratory at GAISh, he had no warm coat and wore a suit and sweaters. His wife, who shared her husband's principles and views, was very kind and considerate to all the family members and their frequent guests. She endured courageously all the life's misfortunes and disfavors. Once, when M. Molodensky first got the State Prize of the USSR, and newspaper correspondents came to take photographs of him, they noticed that he was not wearing a tie. When they asked him to put it on, he refused to do this, saying that he did not want to look unusual. However, in the photographs that appeared next day in newspapers Molodensky was wearing the tie! As it turned out, the correspondents had drawn or painted the tie, but nobody ever guessed it.

Nevertheless, M. Molodensky neither behaved nor looked strangely. On the contrary, he was simple and sociable, but not very verbose, he was very witty and knew many funny anecdotes. His erudition, wittiness and intelligence attracted people, so, when talking he immediately found himself in the center of attention. But when visiting museums or theatres, he grew silent and reserved, absorbed in his thoughts. He was fond of painting, and was interested not only in classic but also in contemporary art. While visiting picture galleries, he paid attention to pictures that were usually passed by the people. Molodensky was hard–working and did not tolerate any casual or indifferent attitude towards work. His room was crowded with colleagues who used to get together there to listen to Molodensky speaking about his latest researches and ideas, to discuss possible variants and directions of them, etc. There were hot disputes and debates, exciting discussions and talks, various and different in their subjects. In addition to these discussions at home, M. Molodensky regularly held informal seminars in his laboratory in pre–war years, when each of the group told about some scientific and technical news he had read during the week, or shared his own ideas. It was very interesting and very useful, as it made all the research fellows keep up with science news in Russian and foreign publications, encouraged them and created a good research team, where there were mutual assistance, understanding, and community of interests and objectives.

It was very interesting to watch Molodensky working when he was on an expedition. He would work for 18 or 19 hours a day without having a break for a meal or for sleep, unless someone pulled him away from the instrument.

It was indeed surprising how he, so busy in his work, managed to be well–informed of all important news concerning his field and related sciences, appreciating properly the results of others' researches and forming his own profound opinion. He was not quick in making any conclusions but was very firm in defending his point of view once formed. He always succeeded in convincing other people of his opinion. It happened so, for example, when he convinced GUGK to change to the system of normal heights, or when he persuaded TsNIIGAIK to take a responsibility for gravimetric survey of the world ocean, and then when he persuaded both GUGK and TsNIIGAiK to change from a gravimetric control network of the highest order of accuracy to a pendulum reference frame, an unprecedented change in modern reference frames.

Molodensky's close relations with TsNIIGAiK for 25 years were interrupted when he was promoted to the position of director of the Institute of Physics of the Earth at the USSR Academy of Sciences, against his will, but at the initiative of the Central Committee of the Communist Party, in spite of the fact that he was not a Communist party member. It was hard to find any other job so unsuitable for Molodensky, as it was completely incompatible with his inclinations and peculiarities of his nature. He always avoided being involved in any administrative activity. He even refused to be a head of the laboratory where he worked, as he did not want to spend his valuable time sitting at meetings, political lessons and seminars. It was impossible to argue with the Central Committee of the Communist Party, and Molodensky gave in. As a director, he did many good things for the Institute and its departments in a short period of time, but a terrible disease was destroying his creative life.

He died on the 12th of November, 1991.

The name of Michail Sergeevich Molodensky has forever entered the history of the Sciences of the Earth.

Эта статья даст Вам общее представление о моих занятиях последнего времени. Ваши замечания приму с благодарностью. ⟶

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ЗАВИСИМОСТЬ ГРАВИТАЦИОННОГО ПОЛЯ ЗЕМЛИ ОТ ИЗМЕНЕНИЯ СКОРОСТИ ЕЕ ВРАЩЕНИЯ

Принято считать, что изменение скорости ω вращения Земли приводит к изменению земного сжатия и среднего значения силы тяжести на земной поверхности. Основы этой теории заложены Клеро, развита она главным образом Ляпуновым.

Более общий подход к той же задаче изложен Л. Лихтенштейном [1]. Он исходил из начальной конфигурации и скорости ω_1 вращения, принимая, что современное состояние Земли близко к равновесному. Этот факт надежно подтверждается астрономическими, геодезическими и геофизическими наблюдениями и положен в основу геологических представлений о строении Земли. Напряжения сдвига внутри Земли рассасываются (релаксируют), поэтому интенсивность современных землетрясений мала по сравнению с той, которая была бы при отсутствии релаксации. Лихтеншейн поставил вопрос об изменении гравитационного поля Земли при малом медленном изменении скорости вращения до значения ω_2. При этом происходит изменение состояния Земли от первого равновесного до второго, тоже равновесного, сила тяжести изменяется в точках поверхности на малую величину δg. Ее нужно определить в зависимости от распределения силы тяжести на поверхности Земли и в соответствии с предположением о состоянии равновесия внутри Земли. Для решения этой задачи Лихтенштейн получил интегральное уравнение, которое до сих пор почти не использовано. Его результат изложен в монографии математического, а не геофизического характера.

Рассмотрев движение планетарной жидкой массы, подверженной только силе взаимного тяготения и одинаковому давлению на поверхности, Пицетти [2] отметил, что единственное движение, как твердого тела, в этом случае есть равномерное вращение около неизменной оси. Изменение угловой скорости ведет к нарушению состояния равновесия и возникновению течений вещества внутри планеты, что в свою очередь может вызывать и поддерживать как горизонтальные, так и вертикальные движения точек земной поверхности.

Мне кажется полезным изложить свои взгляды на пути решения задачи Лихтенштейна исходя из общей теории упругих колебаний Земли.

При изменении скорости вращения внутри Земли (от поверхности до центра) возникают сложные движения, только часть которых можно представить как изменение угловой скорости, т. е. как вращение твердого тела. Эти движения ведут к изменению момента инерции Земли относительно оси вращения или проекции кинетического момента на ту же ось, по какой бы причине такое изменение ни происходило. Смещения, описываемые вращением твердого тела, будут полностью учтены, если уравнения теории упругости составлены в подвижной системе координат, которая вращается с угловой скоростью ω, зависящей от времени, с частотой σ производной ω по времени вокруг оси, мало меняющей направление в пространстве. При известном характере этого движения в уравнения колебаний входят только параметры, полностью определяющие движение. При изменении модуля вектора $\bar{\omega}$ угловой скорости этим параметром является $\dot{\omega}$ — производная от угловой скорости по времени; при изменении направления вектора $\bar{\omega}$ в пространстве (нутации) — компоненты этого вектора по двум координатным осям. Эти величины можно найти из дополнительных условий, определяющих выбор подвижной системы координат. Этот простейший эффект усложнен наложением на него упругих смещений внутри Земли, более сложных, но вполне закономерных, подчиненных закону Гука.

Поэтому возникает задача о колебаниях и изменении гравитационного поля упругой Земли, строение которой в каждой точке определено значениями плотности ρ и модулей упругости μ и k (μ — модуль сдвига; k — модуль всестороннего сжатия). В отдельных полостях возможны значения $\mu=0$, т. е. жидкое состояние. Задача значительно упрощается, так как форма и строение Земли близки к сферически симметричным, т. е. в хорошем приближении зависят только от расстояния r до центра инерции Земли. Необходимо принять во внимание огромные силы взаимного притяжения частиц (гравитационные силы), малую центробежную силу, играющую в этой задаче основную роль, и переносные ускорения (силы Кориолиса) — основные при $\mu=0$ и малой частоте колебания.

С формальной стороны здесь возникает граничная (краевая) задача, подобная хорошо известной геодезистам краевой задаче, связанной с определением фигуры и внешнего гравитационного поля Земли по наблюдениям силы тяжести и другим геодезическим измерениям. Теперь краевая задача связана с более сложными, чем уравнение Лапласа, дифференциальными уравнениями упругости. Радикальное упрощение задачи возникает из-за независимости или слабой зависимости граничных условий от времени. Вращение Земли входит в уравнение и граничные условия через параметры. Сначала следует решить дифференциальные уравнения колебаний при простейших граничных условиях, а на основе этих решений найти частоты колебаний, возможных при отсутствии источников возбуждения (частоты свободных колебаний), и соответствующие им упругие смещения. После этого для каждой частоты находят взаимно независимые свободные колебания, которые могут происходить одновременно. Наконец, следует найти колебания, согласные со всеми граничными условиями на заданной частоте σ.

В целом задача много проще, чем это может показаться на первый взгляд, так как характер движений внутри Земли при $\partial\omega/\partial t=0$ (t — время) довольно хорошо изучен. Известно, что колебания с малыми периодами быстро затухают, а охватывающие всю Землю (прецессия, нутация, вековое замедление земного вращения, смещения Северного и Южного полюсов) выделены и достаточно изучены.

Изменение абсолютной величины ω мало. Давно известно медленное (вековое) изменение, вызываемое лунными приливами в океанах (особенно в мелководье) и приводящее к медленному удалению Луны от Земли. Кроме того, внутреннее строение Земли (функции ρ, μ, k) в течение последних 10—20 лет детально изучено. Точность измерений времени, силы тяжести, деформаций земной поверхности возросла в сотни раз. Современный обзор состояния измерительной техники, результатов наблюдений и теории содержится в книге [3].

Если внутри Земли возникают движения, то только некоторые из них могут повлиять на изменение вращения ее в целом как твердого тела (такое вращение не приводит к упругим напряжениям внутри Земли). Отдельно должны быть рассмотрены движения, происходящие вблизи поверхности Земли и подчиненные более сложным закономерностям (осадки, разрушение гор и вынос продуктов разрушения реками на дно морей и океанов, сезонные изменения в циркуляции атмосферы, а также циклоны, тайфуны, ураганы, приливы в океанах, землетрясения, результаты деятельности человека и т. д.). Внутри же Земли нужно выделить наиболее крупные глубинные землетрясения, связанные с источниками меняющихся со временем упругих напряжений. Исключив эти явления, изучаемые отдельно, движения внутри Земли можно представить уравнениями теории упругости с указанными выше дополнениями и упрощениями. В эти уравнения должны входить параметры, допускающие возможность вращения Земли как твердого тела, т. е. без упругих напряжений. При заданном строении Земли искомое сложное упругое колебание нужно представить суммой простейших гармонических колебаний, каждое из которых может происходить независимо от других. Нужно определить внешние силы, вызывающие колебание (силы притяжения Луны и Солнца и поверхностные силы — давление на поверхность Земли, тангенциальные силы, возникающие от ветровой нагрузки на горы, леса, поля). Все эти воздействия на Землю входят в решение задачи через граничные условия.

Граничные условия можно разложить в ряд, состоящий из слагаемых типа

$$\tau_{nm\sigma} = P_n^m(\cos\vartheta)\cos(\sigma t - m\varphi),$$

где P_n^m — присоединенный полином Лежандра степени n и порядка m; ϑ — полярное расстояние; σ — частота колебания; φ — долгота.

Поэтому каждое простейшее и независимое колебание содержит множитель $\tau_{nm\sigma}$. Когда свободные колебания известны, колебания на любой частоте σ можно представить суммой колебаний на частоте $\sigma=0$ и на частотах свободных колебаний. Таким образом, не требуется интегрировать уравнение на всех частотах (от нуля до бесконечности, хотя в решение в общем случае войдут все частоты). При заданных значениях σ, n, m и $\mu=0$ (неоднородная сжимаемая жидкость) уравнения в частных производных заменяются системой обыкновенных дифференциальных уравнений 4-го порядка при четырех граничных условиях. Этот результат является основным.

Таким образом, значительно упрощается самая трудоемкая работа, а именно, определение собственных частот и функций. В граничные условия, как и в условия непрерывности решения при разрыве ρ, μ и k, входит уравнение поверхности разрыва. Упрощение достигается выбором типа искомого решения, при котором не приходится вычислять интегралы произведения трех сферических функций.

Зависимость ρ, μ и k от широты и долготы выражена слабо, но ее нужно учитывать добавлением малых возмущений. Комплексное значение частоты σ вводит в рассмотрение затухание колебаний.

Собственные частоты и функции определяются с помощью числовых методов. Результаты можно использовать для выделения той части изменения гравитационного поля, которую можно объяснить изменением скорости вращения Земли. Интересно также полученное решение для сравнения ввести в уравнение Лихтенштейна. Рассмотренная здесь задача может служить примером тех задач, которые встанут перед геодезистами, когда от карт изменений высот и силы тяжести во времени они перейдут к геофизическому истолкованию этих результатов.

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ПЕРСПЕКТИВЫ РАЗВИТИЯ ГГС СССР

Обсуждение вопросов классификации государственной геодезической сети (ГГС) СССР направлено в основном на принципы планирования перспективного развития геодезии на основе имеющейся технической базы, которая отчасти отстает от достигнутого уровня в этой области в наиболее развитых

M. S. Molodensky

The dependence of the gravitational field of the Earth on a change of its velocity of rotation

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It is known that a change of the velocity ω of the Earth's rotation leads to a change in the Earth's flattening and in the mean value of gravity on the Earth's surface. The principles of this theory were laid by Clairaut, and it was developed mainly by Liapounov.

L. Lichtenstein 1933 described a more general approach to this problem. He proceeded from an initial configuration and a velocity ω_1 of rotation, supposing that the present state of the Earth is near to equilibrium. This fact is reliably confirmed by astronomic, geodetic and geophysical observations and is taken as a foundation of geological conceptions on the Earth structure. Strains of a displacement relax in the interior of the Earth. Hence the intensity of contemporary earthquakes is small in comparison to what might be without relaxation. Lichtenstein posed the question about a change of the Earth gravitation field by a small slow change of the rotational velocity to a value ω_2. A change of the state of the Earth is taking place from the first equilibrium state to the second state which is likewise a state of equilibrium. Correspondingly, gravity will change at the points of the surface by a small value δg. It must be determined by the dependence of the distribution of gravity on the Earth surface in accordance with the presupposition about preserving the state of equilibrium inside the Earth. Lichtenstein found an integral equation for solving this problem, but this equation was not used so far practically. His result is described in a monograph not of a geophysical but of a mathematical character.

After studying a movement of a planetary fluid mass under the action of the force of mutual attraction and of constant pressure on the surface, Pizzetti 1913 noted that the movement as a rigid body only is a rotation with a constant velocity around an invariable axis. A change of the angular velocity breaks the state of equilibrium and generates currents of matter inside the planet, which can in its turn provoke and support both horizontal and vertical movements of points of the Earth surface.

It seems useful to describe my opinion on methods of a solution of Lichtenstein's problem on the basis of the general theory of elastic oscillations of the Earth.

If the velocity of rotation is changing, complicated movements arise inside the Earth, and only part of these movements can be represented as a change of the angular velocity, i.e. as a rotation of a rigid body. These movements lead to a change of the Earth's moment of inertia with respect to the axis of rotation or a change of a projection of the angular momentum on the same axis, whatever cause had provoked such a change. Displacements capable of being described by a rigid–body rotation, will be taken into account completely by referring the equations of elasticity to a mobile coordinate system which rotates with an angular velocity ω. If this velocity is dependent on time, the derivative of velocity ω, with respect to time, has a frequency σ, and the axis of rotation changes its direction in space only little. When this movement is known, only parameters determining the movement completely will enter into the equations of oscillation. If the magnitude of the vector $\overline{\omega}$ of angular velocity changes, then this parameter is a derivative $\dot{\omega}$ of the angular velocity with respect to time; if the vector $\overline{\omega}$ changes its direction in the space (nutation), then there are components of this vector with respect to two coordinate axes. These values can be found from additional conditions which determine the choice of the mobile coordinate system. This simple effect is complicated by imposing elastic displacements inside the Earth. They are more complex but entirely regular, subject to Hooke's law.

Therefore the following problem arises: To determine the oscillations and the change of the gravitational field of the elastic Earth for a structure which is defined by given values, at every point, of the density ρ and the moduli of elasticity μ and K (μ is the rigidity, the modulus of the shear, K is the modulus of the uniform compression). Values μ=0, i.e. the fluid state, are possible in separate cavities. The problem will be simplified significantly because the form and structure of the Earth are close to spherical symmetry, i.e., to a good approximation, they depend only on the distance from the center of the Earth. It is necessary to take into account the enormous forces of the mutual attraction of particles (the gravitational forces), the small centrifugal force which plays a basic role in this problem, and convective accelerations (Coriolis forces); they are basic for μ=0 and a small frequency of oscillation.

From a formal standpoint there arises a boundary problem; it is similar to the boundary problem which is well known to geodesists, and is connected with the determination of the figure and the exterior gravitational field of the Earth on the basis of gravity observations and other geodetic measurements. The boundary problem is now described by differential equations of elasticity which are more complicated than the Laplace equation. The problem can be simplified radically because of an independence or only a weak dependence of the boundary conditions on time. The rotation of the Earth enters into the equations and boundary conditions through parameters. One should first solve the differential equations of oscillations with the simplest boundary conditions. On the basis of these solutions, one should find frequencies of oscillations which are possible if sources of the excitation (frequencies of free oscillations) are absent, and find elastic displacements corresponding to the frequencies. Then for each frequency it is possible to find mutually independent free oscillations which can occur simultaneously. Finally one should find oscillations consistent with all boundary conditions for the frequency σ under consideration.

On the whole, the problem is considerably easier than it might appear at first sight, as the character of movements inside the Earth with $\partial\omega/\partial t$ = 0 (t denotes the time) has been studied sufficiently well. It is known that oscillations with small periods fade out swiftly, and oscillations which affect the whole Earth (precession, nutation, secular retardation of rotation of the Earth, displacements of the North and South poles) are known adequately.

Changes of the absolute value of the angular velocity ω of the Earth's rotation are small. A slow (secular) change has been known long ago: it is caused by lunar tides in oceans (in shallow water especially) and leads to slow recession of the Moon from the Earth. Furthermore the Earth interior structure (the functions ρ, μ, K) have been studied in detail in the course of the last 10–20 years. The accuracy of measurements of time, gravity, and Earth crust deformations has increased by factors of hundreds. A contemporary review of the state of measurement techniques, observational results and of the theory can be found in Moritz' and Mueller's book 1987.

If movements occur inside the Earth, only some of them can cause a change of its rotation as a rigid body (such a rotation does not lead to elastic stresses inside the Earth). Movements which proceed near the Earth surface must be studied separately; they give rise to more complex interrelations (precipitation, mountain erosion and carrying the products of erosion by rivers to the bottoms of lakes and oceans, seasonal changes in atmospheric circulation and cyclones, typhoons, hurricanes, ocean tides, earthquakes, results of human activity etc.). But inside the Earth we must recognize very deep earthquakes and corresponding elastic stresses changing with time. When these effects will be excluded (they must be studied separately), movements inside the Earth can be represented by equations of the theory of elasticity with additions and simplifications indicated above. Parameters which admit a possibility of rotation of the Earth as a rigid body, i.e. without elastic stresses, must enter into these equations. For the given structure of the Earth, the desired complex elastic oscillations may be represented by a sum of simple harmonic oscillations, and each of the oscillations can be taken into consideration independently of the others.

Exterior forces which cause oscillations (attracting forces of Moon and Sun, as well as superficial forces, pressure on the Earth surface, tangential forces which are caused by wind loading on mountains, forests, fields) must be determined. All these influences on the Earth enter into the solution of the problem through boundary conditions.

The boundary conditions can be developed into a series of terms of type

$$\tau_{nm\sigma} = P_n^m(\cos\theta)\cos(\sigma t - m\phi),$$

where P_n^m is an associated Legendre polynomial of degree n and order m, θ is the polar distance, σ is the frequency of an oscillation, and ϕ is the longitude.

Therefore each simple and independent oscillation contains the factor $\tau_{nm\sigma}$. When the free oscillations are known, the oscillations of any frequency σ can be represented by a sum of oscillations of frequency σ =0 and of the frequencies of the free oscillations. In this way it is not necessary to integrate directly the equations for all frequencies (from zero to infinity), though all frequencies will enter the solution in the general case. Partial differential equations will be replaced, for given values σ, *n, m*, and μ=0 (inhomogeneous compressible fluid), by a system of ordinary differential equations of the fourth order with four boundary conditions. This result is of principal importance.

Thus, the work which demands very much labor, namely the determination of proper frequencies and functions, will be significantly simplified. An equation of discontinuity surfaces enters into the boundary conditions and into the conditions of continuity of the solution at discontinuities of the values ρ, μ and K. A simplification can be attained by a choice of the type of the solution sought for, so that the integrals of products of three spherical functions can be avoided.

The values ρ, μ and K depend on the latitude and longitude weakly, but these dependencies must be taken into account by adding small disturbances. Complex values of the frequency σ introduce a damping of oscillations.

Proper frequencies and functions can be obtained by means of numerical methods. The results can be used for separating the part of the gravitational field which can be explained by a change of the velocity of Earth rotation. For comparison it is interesting to introduce the obtained solution in Lichtenstein's equations. The problem which is described here can be an example of problems which geodesists may have to face when they will proceed from maps of changes of heights and gravity with time to a geophysical interpretation of these results.

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M.S. Molodensky

Equations of a type to which belongs Clairaut's differential equation

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Clairaut's differential equation which connects the flattening of level surfaces of the potential of the gravity with its value on this surface, marked the beginning of physical geodesy. To construct maps representing the Earth surface, geodesists began to measure values of gravity in addition to lengths of lines and angles between these lines. There appeared the necessity to measure gravity on all continents and oceans. Methods of measurements were improved and new methods for land, sea and air measurements were developed. The possibility of search for useful minerals, which can be found because of their influence on gravity, contributed particularly to such a development. Expansions of the gravity potential with respect to spherical harmonics to degree and order 180 was determined by means of the movement of artificial satellites.

Clairaut's differential equation determines the principal part of gravity changes at all depths down to the Earth's center. The physical foundation of this theory is related to the notion that a body, even of the most durable material, becomes fluid by very high pressures and temperatures existent inside the Earth. Shear stresses, which distinguish a solid body from a fluid one, relax rapidly at great depths by dissolving. The relaxation proceeds much more slowly at small depths, as shown by earthquakes. Is it possible to obtain more detailed conclusions about changes of the gravitational field inside the Earth from Clairaut's initial ideas? I shall try to answer this question in what follows.

Any displacements of masses inside the Earth provoke oscillations. They are determined for an elastic Earth by means of differential equations of elastic oscillations and boundary conditions, which depend on stresses at the boundary of an elastic body. The structure of an elastic Earth can be defined by values of the density ρ and the moduli of elasticity K and μ (K is the modulus of the compression, μ is the rigidity). Experimental results and theory lead to the conclusion that the modulus of compression K is not relaxing and that the rigidity μ can relax completely. Then the equations of elastic oscillations will be simplified: they become equations of small oscillations of a fluid. A further simplification of the problem for the Earth is based on closeness of the Earth's structure to a spherical symmetric configuration. This means that the moduli ρ, K and μ can be taken as constant on surfaces of spheres with centers at the Earth's center of inertia as an initial approximation. The boundary surface is spherical in the initial approximation; the boundary conditions do not contradict the differential equations. All functions, which determine displacements and stresses in the elastic Earth, behave as follows: by continuous transition from finite values of μ to zero, they pass continuously into functions which determine small oscillations of the fluid Earth. But Coriolis forces have a basic influence upon oscillations of fluid on low frequencies in the rotating Earth, therefore these forces must enter into the equations of elasticity. In the equations, of course, there must be also included the great forces of the mutual gravitational attraction of the Earth particles and the enormous pressures inside the Earth. Two boundary conditions remain provided that shear stresses are relaxing completely: the first of these conditions determines the pressure at all points of the boundary surface, the second represents the attracting force. Differential equations and boundary conditions of the elastic oscillations admit a continuous transition in the limit to zero values of the rigidity (provided $\sigma^2\rho/\mu$ does not remain finite, where σ denotes the frequency of oscillation).

The partial differential equations of oscillations can be brought by $\mu = 0$ to the form

$$\Delta R = 4\pi G(\rho\Lambda + \rho' H) \quad ; \tag{1}$$

$$(4\omega^2 - \sigma^2)\psi = R + w' H + \frac{1}{\rho} K\Lambda \tag{2}$$

$$\Delta\psi = \chi' r + 2\chi \tag{3}$$

$$(4\omega^2 - \sigma^2)\chi r^2 = \left(w'\rho - \frac{\rho'}{\rho} K \right)\Lambda \tag{4}$$

where Δ denotes Laplace's operator, R a change of the gravity potential by a deformation of the Earth, G the gravitational constant, Λ the cubical dilatation, H a radial displacement, ω the angular velocity of the Earth's rotation, σ the frequency of an oscillation; ψ, χ functions which determine components of a displacement by deformation, r the distance from the Earth's center, and w is the gravity potential; a prime denotes differentiation with respect to the distance r.

The expressions are obtained for H_{nm} and Λ_{nm} (n is the degree of a harmonic, m its order):

$$H_{nm} = \varepsilon_{nm}\left(\psi' - r\chi - \frac{F}{\varepsilon r}\psi \right)_{nm} \quad ;$$

$$\Lambda_{nm} = \varepsilon_{nm}\Delta\psi_{nm} \quad ,$$

where for $\sigma^2 << \omega^2$

$$\varepsilon_{nm} = -(C_{nm} + C_{n+1,m}) \quad ;$$

$$F_{nm} = (n+1)C_{nm} - nC_{n+1,m} \quad ;$$

$$C_{nm} = \frac{4\omega^2}{\sigma^2} \cdot \frac{n^2 - m^2}{4n^2 - 1} \quad .$$

A solution is possible for all values n and m, provided factors of ε_{nm} reduce to zero. Hence the values χ_{nm} and ψ_{nm} can be determined:

$$\psi_{nm} = C_1 r^{p} + C_2 r^{-p-1} \quad ;$$
$$\Delta\psi_{nm} = \left(C_1 r^{p-2} + C_2 r^{-p-3}\right)\left(p(p+1) - n(n+1)\right) \; .$$

From the condition of boundedness (regularity) of the value ψ_{nm} at the Earth center it follows that either $C_1 = 0$ or $C_2 = 0$. Then a regular solution for the function χ_{nm} can be found.

The simplest solution of these equations, by the spherical symmetric structure of the Earth, has the form

$$\begin{aligned} \Phi &= \Phi_{nm}(r)\tau_{nm} \quad ; \\ \tau_{nm} &= P_n^m(\cos\theta)\cos(\sigma t - m\phi) \end{aligned} \quad , \tag{5}$$

where Φ is one of the functions R, H, Λ, ψ or χ; *P denotes* Legendre's polynomial, θ polar distance, t is time, and ϕ is longitude.

If the perturbations of the spherical symmetry of the Earth are small, the functions K and ρ can be represented by sums

$$K = K(r) + \sum\left(a_{nm}\cos m\phi + b_{nm}\sin m\phi\right)P_n^m(\theta) \quad ;$$
$$\rho = \rho(r) + \sum\left(c_{nm}\cos m\phi + d_{nm}\sin m\phi\right)P_n^m(\theta)$$

with small coefficients, a_{nm}, b_{nm}, c_{nm}, d_{nm}. Then a solution of the equations (1) – (4) can be obtained in the simple form (5). The functions χ, H and Λ are determined through ψ. After the functions H and Λ are excluded from (1) by means of (2) and (4), we get

$$\Delta R + 4\pi G\frac{\rho'}{w'}R = 4\pi G\left(\sigma^2 - 4\omega^2\right)\left(\frac{\rho'}{w'}\psi + \frac{r^2}{w' - \left(\rho'/\rho^2\right)K}\chi\right) \; . \tag{6}$$

This equation, by the spherically symmetric structure of the Earth for $\psi = \chi = 0$, $n = 2$, $m = 0$ turns into Clairaut's well–known differential equation.

The boundary conditions are determined by the superficial and mass forces acting on the Earth: by the pressure on the boundary surface and by the attraction potential of the deformable Earth. If the boundary conditions are homogeneous, as the differential equations are, then the solution $R = H = \Lambda = \psi = \chi = 0$ is possible. However, potential R and the potential of masses which disturb the symmetry of the Earth structure, become more and more indivisible by the frequency σ tending to zero. It would be better to include relaxing rigidity, which depends on time, in the boundary conditions and to use Fourier's integral transformation, which introduces in the problem frequency characteristics of the relaxation process. Instead of such an approach it can be assumed that the potential R on the boundary surface for $\sigma = 0$ is the same as the known real Earth potential. At first sight it seems that such a strong condition is impossible. However, the arbitrary constants of integration for different n and m can be expressed through the coefficients of expansion of the Earth's potential into series of the spherical functions, and the boundary conditions become inhomogeneous. It is also interesting which form the level surface will get at the boundary of the Earth's core. Perfecting the theory of the oscillations and rotation of the Earth to meet the accuracy of the new geodetic techniques will demand considerable efforts.

Molodensky's Theory and GPS

Helmut Moritz

1. Introduction

As shown in the article by Brovar and Yurkina in this volume, Molodensky's theory is a completely new approach, a change of perspective, even a "Copernican revolution" in geodesy. The focus of attention is shifted from the classical geoid to a direct determination of the physical Earth surface. As a matter of fact, this involves a special difficult boundary value problem, "Molodensky's problem", which was formulated by M.S. Molodensky in terms of an integral equation and solved by a "Molodensky series".

The importance of Molodensky's theory consists in the approach, the emphasis on the basic importance on the physical Earth surface *S*. Today, *S* can be directly geometrically determined using GPS, the Global Positioning System, and similar satellite techniques (GLONASS). This permits us to use Molodensky's approach to the direct determination of the potential *W*, replacing the tedious and time–consuming operation of leveling. The (basically very simple) mathematical approach to Molodensky's determination of *S* and the new determination of *W*, which permits a presentation from a common point of view, will be outlined in sec. 2.

2. Geodetic Boundary–Value Problems

In space we have the well–known fact that the gravity vector $\underline{g}$ and the gravity potential (geopotential) *W* are related by

$$\underline{g} = \text{grad}\, W \equiv \left(\frac{\partial W}{\partial x}, \frac{\partial W}{\partial y}, \frac{\partial W}{\partial z} \right) \quad , \tag{1}$$

that is the force $\underline{g}$ is the gradient vector of the potential.

Let *S* be the Earth's topographic surface and let *W* and $\underline{g}$ be the geopotential and the gravity vector *on this surface* (Fig. 1). Then there exists a relation

$$\underline{g} = f(S, W) \quad , \tag{2}$$

the gravity vector $\underline{g}$ on *S* is a function of the surface *S* and the geopotential *W* on it.

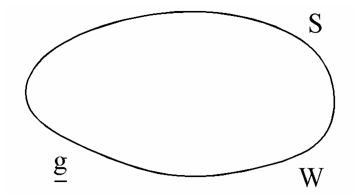

Figure 1. The geopotential *W* and gravity $\underline{g}$ are defined in the Earth's surface *S*

This can be seen in the following way. Let the surface *S* and the geopotential *W* on *S* be given. The gravitational potential *V* is obtained by subtracting the potential of the centrifugal force Φ, which is simple and perfectly known:

$$V = W - \Phi \quad , \tag{3}$$

cf. (Heiskanen and Moritz 1967, sec. 2–1). (This readily available book will in the following simply be denoted by PG, "Physical Geodesy".)

The potential *V* outside the Earth is a solution of Laplace's equation $\Delta V = 0$ and consequently *harmonic* (PG sec. 1–2). Thus knowing *V on S* we can obtain *V outside S* by solving Dirichlet's boundary–value problem, the first boundary–value problem of potential theory, which is always uniquely solvable (PG sec. 1–7) (at least if *V* is sufficiently smooth on *S*). After having found *V* as a function in space outside *S*, we obtain the gravitational force grad*V*. Adding the well–known and simple vector of the centrifugal force, we obtain the gravity vector $\underline{g}$ outside and, by continuity, on *S*.

This is precisely what (2) means. The modern general concept of a function can be explained as a *rule of computation*, indicating that given *S* and *W* on *S*, we can uniquely calculate $\underline{g}$ on *S*. (Of course, *f* is not a function in the elementary sense, but rather a "non–linear operator", but we shall disregard this for the moment.)

(1) Molodensky's boundary–value problem is the task to determine the Earth's surface *S* if $\underline{g}$ and *W* on it are given. Formally we have to solve (2) for $\underline{S}$:

$$S = F_1(\underline{g}, W) \quad , \tag{4}$$

that is, we get *geometry from gravity*.

(2) GPS boundary–value problem. Since we now have GPS at our disposal, we can consider *S* as known, or at least determinable by GPS. In this case, the *geometry* (*S*) is known, and we can solve (2) for *W* :

$$W = F_2(S, \underline{g}) \quad , \tag{5}$$

that is, we get *potential from gravity*. As we shall see, this is far from trivial: we have now a method to *replace leveling by GPS*, a tedious and time–consuming old–fashioned method by a fast modern technique.

In spite of all similarity we should bear in mind a fundamental difference: (5) solves a *fixed boundary problem* (boundary *S* given) whereas (4) solves a *free boundary problem* : the boundary *S* is a priori unknown (“free”). Fixed boundary problems are usually simpler than free ones.

This is only the principle of both solutions. The direct implementation of these formulas is difficult because that would imply the solution of “hard inverse function theorems” of nonlinear functional analysis. For numerical computations, we know series solutions, in the form of “Molodensky series”, which are sufficient for all present purposes and which can, furthermore, be derived in an elementary fashion, without needing integral equations (Molodenski 1958; Molodenskii et al. 1962; PG chapter 8; APG sec. 45). (APG stands for “Advanced Physical Geodesy” (Moritz 1980).) Here we shall outline the known elementary solution for Molodensky’s problem and immediately extend it to the GPS problem. Both problems will be solved by very similar Molodensky series.

3. Linearization

We recall the linearization for the Molodensky problem, which applies to the GPS problem as well.

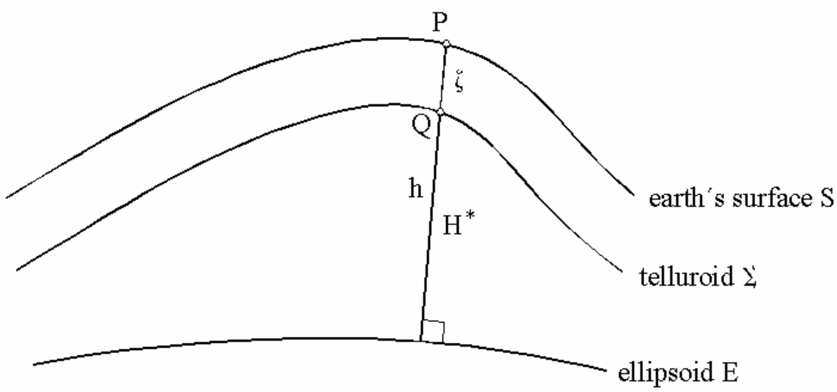

Fig. 2. The telluroid. The normal height *H** and the height anomaly ζ

The geometry is familiar (Fig. 2). In much the same way as the ellipsoid corresponds to the geoid, we construct a surface Σ, the *telluroid*, which is defined by the condition

$$U(Q) = W(P) \quad , \tag{6}$$

the normal potential *U* at any telluroid point *Q* is equal to the geopotential *W* at the surface point *P* situated along the same ellipsoidal normal. The normal potential *U* is the potential of the reference ellipsoid; the normal gravity vector $\underline{\gamma} = \text{grad}U$ and normal gravity $\gamma = ||\underline{\gamma}||$ are referred to *U*, in the same way as $\underline{g}$ and g are referred to *W*.

Note that (6) is the surface equivalent to the classical relation for sea level (Fig. 3)

$$U(Q_0) = W(P_0) \quad . \tag{7}$$

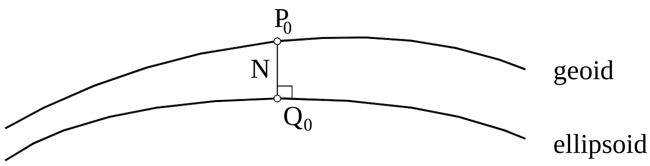

Fig. 3. Sea level: geoid and ellipsoid

Eq. (7) would hold, with

$$W(P_0) = W_0 = \text{const.} \quad , \tag{8}$$

If *S* were an equipotential surface, the *geoid,* which is the case only over the oceans (with the usual simplifying assumption that the surface of the ocean is an equipotential surface not changing with time) (Fig.3).

Of course, Molodensky's theory directly uses not the geoid but the physical Earth surface. (We repeat once more that this in Molodensky's epochal idea which radically changed the course of physical geodesy since 1945.)

We do, however, use the fictitious case of *S* being an equipotential surface, but only as a first (or zero–order) assumption in a perturbation approach for the real Earth surface (Molodensky series). This first approximation is the *spherical case* to be considered in the next section.

The ellipsoidal height *h* is directly determined by GPS. It may be decomposed as follows

$$h = H^* + \zeta \quad . \tag{9}$$

Here, *H** is the *normal height* and ζ is the *height anomaly*, whose definitions are seen from Fig. 2.

The definition of the *gravity anomaly* Δg and the *gravity disturbance* δg has, on the Earth's surface, the same form as in the classical case of geoid and sea level:

$$\Delta g = g_P - \gamma_Q = -\frac{\partial T}{\partial h} + \frac{1}{\gamma}\frac{\partial \gamma}{\partial h} T \quad , \tag{10}$$

$$\delta g = g_P - \gamma_P = -\frac{\partial T}{\partial h} \quad . \tag{11}$$

The gravity disturbance δg has become practically important only through GPS, since the GPS height *h* of *P* can be measured and hence normal gravity γ at *P*, γ_P, can be determined.

As usual, Bruns' formula holds both at P_0 (classical geoid height *N*) and *P* (Molodensky height anomaly ζ):

$$N = \frac{T(P_0)}{\gamma} \quad , \tag{12}$$

$$\zeta = \frac{T(P)}{\gamma} \quad , \tag{13}$$

with some approximate value for γ. Eq. (10) can be reformulated as the boundary conditions for the Molodensky problem

$$\frac{\partial T}{\partial h} - \frac{1}{\gamma}\frac{\partial \gamma}{\partial h} + \Delta g = 0 \quad , \tag{14}$$

(cf. PG p. 86), and for the GPS problem (PG p. 85)

$$\frac{\partial T}{\partial h} + \delta g = 0 \quad , \tag{15}$$

These two boundary conditions hold both at the surface *S* (Molodensky) and at sea level (PG p. 298).

Finally we introduce the *spherical approximation*, disregarding the flattening *f* in the equations (which are linear relations between *small* quantities). The spherical approximation is a formal operation (disregarding *f* in small *ellipsoidal* quantities) and does not mean using a "reference sphere" instead of a reference ellipsoid in any geometrical sense (APG p. 15). Then (14) and (15) reduce to

$$\frac{\partial T}{\partial r} + \frac{2}{r}T + \Delta g = 0 \quad , \tag{16}$$

$$\frac{\partial T}{\partial r} + \delta g = 0 \quad , \tag{17}$$

These equations, for the Molodensky and the GPS problem, are valid both at sea level (classical, PG p. 88) and at *S* (Molodensky, PG sec. 8–6).

4. The Spherical Case

As we have agreed, we work formally with a sphere (the reference ellipsoid stays at its geometric place!). This means putting *r* = *R* = const. Furthermore we assume (fictitiously!) that *S* is a level surface.

Expanding T and Δg into a series of Laplace spherical harmonics (PG p. 97) we find

$$T(\theta,\lambda) = \sum_{2}^{\infty} T_n(\theta,\lambda) \quad , \tag{18}$$

$$\Delta g(\theta,\lambda) = \sum_{2}^{\infty} \Delta g_n(\theta,\lambda) \quad , \tag{19}$$

on the surface of the sphere, whence by (16) with $r = R$,

$$T = R\sum_{n=2}^{\infty} \frac{\Delta g_n}{n-1} \quad . \tag{20}$$

The summation starts conventionally with n = 2, rather than n = 0, for several reasons, one of them being that n = 1 would lead to a zero denominator in (20).

As indicated in PG, p. 97, this leads to well–known Stokes' formula

$$T = \frac{R}{4\pi}\iint_{\sigma} S(\psi)\Delta g d\sigma \tag{21}$$

where

$$S(\psi) = \sum_{n=2}^{\infty} \frac{2n+1}{n-1} P_n(\cos\psi) \quad , \tag{22}$$

where $P(\cos\psi)$ are Legendre Polynomials. Here ψ denotes this spherical distance from the point at which T is to be computed.

In exactly the same way we obtain for the gravity disturbance with the boundary condition (17)

$$\delta g(\theta,\lambda) = \sum_{0}^{\infty} \delta g_n(\theta,\lambda) \quad , \tag{23}$$

$$T(\theta,\lambda) = R\sum_{n=0}^{\infty} \frac{\delta g_n}{n+1} \quad , \tag{24}$$

and the *formula of Neumann–Koch*

$$T = \frac{R}{4\pi}\iint_{\sigma} K(\psi)\delta g d\sigma \tag{25}$$

with

$$K(\psi)=\sum_{n=0}^{\infty}\frac{2n+1}{n+1}P_n(\cos\psi) \tag{26}$$

and by summation of this series

$$K(\psi)=\frac{1}{\sin(\psi/2)}-\ln\left(1+\frac{1}{\sin(\psi/2)}\right) \tag{27}$$

being the *Neumann–Koch function*.

Mathematically, this is the *solution of Neumann's problem (the second boundary value problem of potential theory) for the sphere*, cf. PG, p. 36. It is a classical problem of potential theory, with a history of at least 150 years, similarly to Stokes' formula. "Neumann's problem" is called after the mathematician Carl Neumann, who edited this father's (Franz Neumann) lectures from the 1850ies (Neumann 1887, see especially p. 275). The external spherical Neumann problem also occurs in (Kellogg 1929, p. 247). Later authors connected with this formula were Bjerknes (the famous Swedish meteorologist or his father, a mathematician) and the Russians Idelson and Malkin, both around 1932. It is again found in Hotine (1969, pp. 311, 318). (I owe these historical remarks to Mrs. M.I. Yurkina.)

Their basic significance for modern physical geodesy with a known Earth surface was recognized and elaborated by Koch (1971).

So in the GPS boundary problem on the sphere, the solution (25) is completely analogous to the formula of Stokes (22).

The fact that the GPS problem is conceptually simpler (fixed boundary surface) than Molodensky's problem (free boundary surface) is expressed by the fact that Stokes' function *must* start with n = 2, since n = 1 gives a zero denominator, whereas Neumann–Koch's function (26) is regular for all n.

In both cases, the height anomaly ζ (here the geoidal height) is given by Bruns' formula

$$\zeta=\frac{T}{\gamma_0}\ , \tag{28}$$

where γ_0 is a suitable mean value of normal gravity (e.g. 980 gal).

We shall see that these spherical solutions form the base for an elementary solution of Molodensky's problem and the GPS problem for the Earth's surface. We only mention the well–known fact that, for the Earth's surface S, these two problems are *oblique–derivative problems*, since the direction of the plumb line does not coincide with the normal to the Earth's surface, at least on land. Thus the GPS boundary problem for S is not a Neumann problem (which always involves the normal derivative)!

5. *Analytical Continuation to a Sphere*

The idea is very simple (Fig. 4). Our observations Δg or δg, given on the Earth's surface *S*, are reduced, or rather *analytically continued* (upward or downward, see Fig. 3), to a level surface (or normal level surface $U = U_P$, which for our purpose is the same). In the spherical approximation both surfaces $U = U_P$ and $U = U_0$, are concentric spheres, but only in the precise sense of the spherical approximation as explained above.

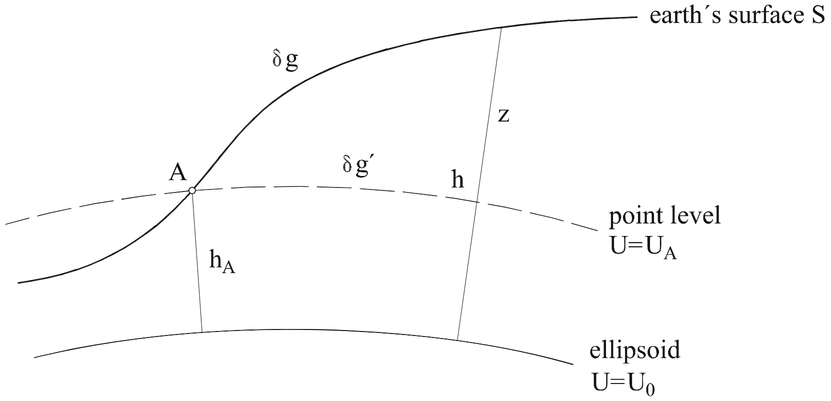

Fig. 4. Analytical continuation from the Earth surface to point level

Following APG, p. 378, we have

$$\Delta g = \Delta g' + z\frac{\partial \Delta g'}{\partial z} + \frac{1}{2!}z^2\frac{\partial^2 \Delta g'}{\partial z^2} + \frac{1}{3!}z^3\frac{\partial^3 \Delta g'}{\partial z^3} + \ldots$$

$$= \Delta g' + \sum_{n=1}^{\infty}\frac{1}{n!}z^n\frac{\partial^n \Delta g'}{\partial z^n} \quad , \qquad (29)$$

where

$$z = h - h_A \qquad (30)$$

is the elevation difference with respect to the computation point *A*. For the present we shall assume the series (29) to be convergent.

Here $\Delta g'$ is the gravity anomaly at point level (Fig. 4). The use of a Taylor series is typical for analytical continuation (cf., for instance, the use of Taylor series for analytical continuation for functions of a complex variable).

What does analytical continuation mean in our context? The external potential *V*, and also the functions $r\Delta g$ and $r\delta g$ (PG p. 90), are *harmonic* outside *S*, and analytical continuation by Taylor series again generates a harmonic function. This harmonic function coincides with the potential *V* outside *S*, but inside *S* it is different from *V* since *V* inside the masses does not satisfy Laplace's equation $\Delta V = 0$. Inside *S*, a Taylor series simply represents the *analytical continuation of V* which, of course, is harmonic.

The use of analytical continuation has an interesting history. It was first considered, as a possibility, by Molodensky himself, already before 1945, but he soon rejected this method! Molodensky was a profound and very serious mathematician, with a high regard for mathematical rigor. He would not be satisfied with intuitive heuristic approaches, so common in mathematical physics.

In fact, the analytical continuation of the external gravitational potential into the interior of the Earth's masses is very likely to become singular at some points. As a serious mathematician, Molodensky rejected the use of singular functions for regular purposes.

Still, analytical continuation continued to exert an irresistable fascination because its use is so easy. It was rediscovered around 1960 by Arne Bjerhammar. At the General Assembly of the International Union of Geodesy and Geophysics in Berkeley, California, in 1963, I talked to Arne about these difficulties, but he refused to take them seriously. So we decided to take a long walk from the Berkeley bus terminal to the Golden Gate Bridge in San Francisco. I was admiring the beautiful scenary, but Arne kept talking about analytical continuation. Finally, tired by the hike, I was ready to admit that analytical continuation was rigorously possible for *discrete* boundary data (all our terrestrial gravity measurements are discrete) and approximately possible for continuous boundary data.

This admittedly intuitive thinking was made rigorous by Krarup's (1969) idea that Runge's theorem, well known for approximation of analytical functions of a complex variable, should be applied to the problem of analytical continuation of harmonic functions in space. Runge's theorem, in the form of Krarup, says, loosely speaking, that, even if the external geopotential cannot be regularly continued from the Earth surface S into its interior, it can be made continuable by an *arbitrarily small* change of the geopotential at S. (For a detailed discussion see APG, secs. 6 to 8.) (Another historical remark: the Krarup-Runge theorem for harmonic functions in space goes back (at least) to Szegö and to Walsh (both around 1929), cf. (Frank–Mises 1937, pp. 760–762). It is always dangerous to talk about priorities!)

So, in the same year 1969, Marych and Moritz independently found an elementary solution by analytical continuation in the form of a Molodensky series as will be shown now. (By the way, analytical continuation is a purely mathematical concept independent of the density of the topographic masses. Thus it is not an "introduction of gravity reduction through the back door", which would be contrary to the spirit of Molodensky's theory.)

Let us return to series (29). It may be written in the symbolic form

$$\Delta g = U \Delta g' \quad , \tag{31}$$

where U is the *analytic continuation operator*. Since we are given $\Delta g'$ at the Earth's surface and wish to compute $\Delta g'$ at the sphere representing point level, we must invert (31):

$$\Delta g' = U^{-1} \Delta g = D \Delta g \quad , \tag{32}$$

where the operator *D* is inverse to *U*. (The notation *U* and *D* has been motivated by *U*pward and *D*ownward continuation but *U* may also be downward as Fig. 4 shows.)

The reader not interested in the details may skip the following derivation and go directly to the computational formulas (64) to (73),

Since $\Delta g'$ refers to a level surface, we may directly apply the usual formulas of Stokes and Vening–Meinesz to the anomalous potential *T* and the deflection of the vertical (ξ, η), all at the point *A* :

$$T = \frac{R}{4\pi} \iint_\sigma \Delta g' S(\psi) d\sigma \quad , \tag{33}$$

$$\begin{Bmatrix} \xi \\ \eta \end{Bmatrix} = \frac{1}{4\pi\gamma^0} \iint_\sigma \Delta g' \frac{dS}{d\psi} \begin{Bmatrix} \cos\alpha \\ \sin\alpha \end{Bmatrix} d\sigma \quad . \tag{34}$$

Strictly speaking, we should in (33) replace R by $R + h_A$, but we may use *R* without impairing the accuracy; in the same way we may in (34) use a mean value γ^0 of about 980 gal as usual.

Derivation of $\Delta g'$. There remains now to compute the point level anomaly $\Delta g'$ from the measured ground anomaly Δg . We write (29) symbolically in the form

$$\Delta g = \Delta g' + \left(\sum_{n=1}^{\infty} \frac{1}{n!} z^n \frac{\partial^n}{\partial z^n} \right) \Delta g'$$

$$= \left(I + \sum_{n=1}^{\infty} z^n L_n \right) \Delta g' \quad ; \tag{35}$$

$$L_n = \frac{1}{n!} \frac{\partial^n}{\partial z^n} = \frac{1}{n!} \frac{\partial^n}{\partial r^n} \tag{36}$$

is a vertical (radial) differentiation operator and *I* is the identity operator:

$$I f = f \quad . \tag{37}$$

Comparing (35) with (31) we see that we have obtained a symbolic series expansion of the analytical continuation operator *U* :

$$U = I + \sum_{n=1}^{\infty} z^n L_n \quad . \tag{38}$$

We shall now try to compute the inverse continuation operator

$$D = U^{-1} \tag{39}$$

by forming the formal reciprocal of the series (38); then (32) gives $\Delta g'$. This will be done as follows.

We replace all elevations *h* by *kh* where *k* is the Molodensky parameter with $0 \le k \le 1$. Then the analytical continuation operator (38) becomes

$$U = I + \sum_{n=1}^{\infty} k^n z^n L_n = \sum_{n=0}^{\infty} k^n U_n \quad , \tag{40}$$

where

$$U_0 = I \quad ; \quad U_n = z^n L_n \quad \text{if} \quad n = 1,2,3, \ldots \tag{41}$$

In the same way we express the downward continuation operator $D = U^{-1}$ as a formal series

$$D = \sum_{n=0}^{\infty} k^n D_n \quad . \tag{42}$$

We may try to determine the D_n from the obvious operator identity

$$U D = I \quad . \tag{43}$$

On substituting the respective series we have

$$\sum_{p=0}^{\infty} k^p U_p \sum_{q=0}^{\infty} k^q D_q = I \tag{44}$$

or

$$\sum_{p=0}^{\infty} \sum_{q=0}^{\infty} k^{p+q} U_p D_q = I \quad . \tag{45}$$

This may be transformed into

$$\sum_{n=0}^{\infty} k^n \sum_{r=0}^{n} U_r D_{n-r} - I = 0 \quad . \tag{46}$$

We require this identity to hold for all values of the parameter *k*. Then the factors of all k^n must be zero. For *n* = 0 we have

$$U_0 D_0 - I = 0 \quad , \tag{47}$$

thus because of $U_0 = I$ also

$$D_0 = I \quad . \tag{48}$$

For $n \neq 0$ we have the equation

$$\sum_{r=0}^{n} U_r D_{n-r} = 0 \tag{49}$$

or

$$D_n + \sum_{r=1}^{n} U_r D_{n-r} = 0 \quad ,$$

whence

$$D_n = -\sum_{r=1}^{n} U_r D_{n-r} \quad . \tag{50}$$

This equation expresses D_n in terms of the known U_r and the previously determined $D_1, D_2, \ldots, D_{n-1}$. So, starting from $D_0 = I$, we can recursively compute the operators $D_1, D_2, D_3, \ldots$

Computationally more convenient is the introduction of the functions

$$g_n = D_n(\Delta g) \quad . \tag{51}$$

Eq. (49) gives for them

$$\sum_{r=0}^{n} U_r D_{n-r}(\Delta g) = 0 \quad . \tag{52}$$

By (41) and (51) this becomes

$$\sum_{r=0}^{n} z^r L_r(g_{n-r}) = 0 \quad , \tag{53}$$

which can be solved for g_n, noting $z^0 L_0(g_n) = g_n$:

$$g_n = -\sum_{r=1}^{n} z^r L_r(g_{n-r}) \quad . \tag{54}$$

Eq. (54) makes it possible to determine the g_n recursively, starting from

$$g_0 = \Delta g \quad . \tag{55}$$

Then the anomaly $\Delta g'$, defined by (32), is then given by

$$\Delta g' = D\Delta g = \sum_{n=0}^{\infty} D_n(\Delta g) = \sum_{n=0}^{\infty} g_n \quad . \tag{56}$$

We have put k = 1 in (42), so as to change kh back into the actual elevation h, since we had admitted a general k only in order to get a convenient mechanism of expansion.

Then (33) gives

$$T = S(\Delta g') = \sum_{n=0}^{\infty} T_n \tag{57}$$

with

$$T_n = S(g_n) \quad , \tag{58}$$

S denoting the Stokes operator.

Determination of the L_n. We must now study the operators L_n which play a basic role in the present method.

First we derive some simple formulas for them. The definition (36) gives

$$L_n = \frac{1}{n!}\frac{\partial^n}{\partial z^n} = \frac{1}{n}\frac{1}{(n-1)!}\frac{\partial^{n-1}}{\partial z^{n-1}}\frac{\partial}{\partial z} \tag{59}$$

or

$$L_n = \frac{1}{n}L_{n-1}L = \frac{1}{n}LL_{n-1} \quad . \tag{60}$$

This is a *recursion formula* expressing L_n in terms of L_{n-1} and $L = L_1$.

Repeated application of this recursion formula gives

$$L_n = \frac{1}{n!}L^n \tag{61}$$

where

$$L^n = LLL\ldots L \qquad (n \text{ times}) \quad ;$$

this is also evident from (59).

The original meaning of L_n as a spatial operator, namely a vertical derivative, is restricted to the use with level–surface anomalies $\Delta g'$ only; furthermore, this vertical derivative is normal to the surface and thus, figuratively speaking, leads out of the surface.

It is possible, however, to interpret L_n as a *surface operator* which does not lead out of the surface and can be used with data given on an arbitrary smooth surface which need not be a level surface. This may be done as follows.

The vertical derivative $\partial/\partial r$ can be expressed in terms of surface values by the well–known spherical formula (PG, p. 38)

$$\frac{\partial f}{\partial r} = -\frac{1}{R} f + \frac{R^2}{2\pi} \iint_\sigma \frac{f - f_P}{l_0^3} d\sigma \quad . \tag{62}$$

P is the point at which $\partial f/\partial r$ is computed and to which f in the first term on the right–hand side refers. σ denotes the unit sphere and

$$l_0 = 2R \sin\frac{\psi}{2} \quad . \tag{63}$$

Computational formulas. Let us finally summarize our computational formulas. By Bruns' equation (13) and by (33), (34), and (56) we have

$$\zeta = \frac{R}{4\pi\gamma^0} \iint_\sigma \Delta g S(\psi) d\sigma + \sum_{n=1}^{\infty} \frac{R}{4\pi\gamma^0} \iint_\sigma g_n S(\psi) d\sigma \quad , \tag{64}$$

$$\begin{Bmatrix} \xi \\ \eta \end{Bmatrix} = \frac{1}{4\pi\gamma^0} \iint_\sigma \Delta g \frac{dS}{d\psi} \begin{Bmatrix} \cos\alpha \\ \sin\alpha \end{Bmatrix} d\sigma + \sum_{n=1}^{\infty} \frac{1}{4\pi\gamma^0} \iint_\sigma g_n \frac{dS}{d\psi} \begin{Bmatrix} \cos\alpha \\ \sin\alpha \end{Bmatrix} d\sigma \quad . \tag{65}$$

Here γ^0 is a global mean value such as 980 gal. The correction terms g_n are evaluated recursively by (54):

$$g_n = -\sum_{r=1}^{n} z^r L_r(g_{n-r}) \quad , \tag{66}$$

starting from

$$g_0 = \Delta g \quad ; \tag{67}$$

there is

$$z = h - h_A \quad . \tag{68}$$

The L_n are also evaluated recursively as *surface operators* :

$$L_n(\Delta g) = \frac{1}{n} L_1\left[L_{n-1}(\Delta g)\right] \tag{69}$$

with the *surface integral*

$$L_1(f) = \frac{R^2}{2\pi} \iint_\sigma \frac{f - f_P}{l_0^3} d\sigma \quad . \tag{70}$$

These formulas are all that is needed to compute approximations of an arbitrarily high order. All occurring operators are systematically reduced to a repeated application of the integral (70).

Let us finally render the method more concrete by evaluating (66) explicitly for n = 1, 2, 3:

$$\begin{aligned} g_1 &= -zL_1(\Delta g) \quad , \\ g_2 &= -zL_1(g_1) - z^2 L_2(\Delta g) \quad , \\ g_3 &= -zL_1(g_2) - z^2 L_2(g_1) - z^3 L_3(\Delta g) \quad . \end{aligned} \tag{71}$$

If we restrict ourselves to n = 1, then the present solution becomes

$$\zeta = \frac{R}{4\pi\gamma^0} \iint_\sigma \left[\Delta g - (h - h_A) \frac{\partial \Delta g}{\partial h} \right] S(\psi) d\sigma \tag{72}$$

since

$$g_1 = -(h - h_A) L_1(\Delta g) = -(h - h_A) \frac{\partial \Delta g}{\partial z} = -(h - h_A) \frac{\partial \Delta g}{\partial h} \tag{73}$$

and the operator L_1 may be interpreted as a vertical derivative by (36). This first–order solution may, therefore, be called *gradient solution*. Analogous formulas hold for ξ and η. All these formulas are very suitable for practical application, cf. PG, secs. 8–8 and 8–9.

This series solution is completely equivalent, term by term, to the usual Molodensky series obtained from an integral equation. This has been proved in an ingenious way by L.P. Pellinen; cf. APG sec. 46 ("Pellinen's equivalence proof"). This is a final confirmation of the validity of the analytical continuation approach.

6. Gravity Disturbances: the GPS Case

The basic fact is that for gravity disturbances the derivation of "Molodensky corrections" g_n is identical to the Δg case. Therefore we have given the derivation from eq. (29) to (63) in full detail, following APG sec. 45 for convenience.

The reason is that the gravity disturbance δg has exactly the same analytical behavior as the gravity anomaly Δg since $r\delta g$, as a function in space, is harmonic together with $r\Delta g$. Thus the arguments leading from eq. (29) to (66) are literally the same, only Δg has to be replaced by δg, and Stokes' formula must be replaced by the formula of Neumann–Koch (25) and similarly for Vening–Meinesz' formula.

Thus we obtain

$$\zeta = \frac{R}{4\pi\gamma^0} \iint_\sigma \delta g K(\psi) d\sigma + \sum_{n=1}^{\infty} \frac{R}{4\pi\gamma^0} \iint_\sigma g_n K(\psi) d\sigma \quad , \tag{74}$$

$$\begin{Bmatrix} \xi \\ \eta \end{Bmatrix} = \frac{1}{4\pi\gamma^0} \iint_\sigma \delta g \frac{dK}{d\psi} \begin{Bmatrix} \cos\alpha \\ \sin\alpha \end{Bmatrix} d\sigma + \sum_{n=1}^{\infty} \frac{1}{4\pi\gamma^0} \iint_\sigma g_n \frac{dK}{d\psi} \begin{Bmatrix} \cos\alpha \\ \sin\alpha \end{Bmatrix} d\sigma \quad . \tag{75}$$

For the “Vening–Meinesz GPS formula” (75) we find by differentiation of (27)

$$\frac{dK}{d\psi} = -\frac{1}{2} \frac{\cos(\psi/2)}{\sin^2(\psi/2)} \frac{1}{1+\sin(\psi/2)} \quad . \tag{76}$$

The correction terms g_n are again evaluated recursively by

$$g_n = -\sum_{r=1}^{n} z^r L_r(g_{n-r}) \quad , \tag{77}$$

but now, of course, we start from

$$g_0 = \delta g \tag{78}$$

Of course, formulas (69) to (73) continue to hold. We only have to replace Δg by δg and $S(\psi)$ by $K(\psi)$.

Let us summarize again our trick for solving the modern boundary–value problems (Molodensky and Koch). It is difficult to directly work with the complicated Earth surface *S*. Therefore, by analytical continuation of Δg or δg, respectively, we *reduce these complicated problems to the corresponding spherical problems*, for which the solution is simple and well known.

The similarity of the Molodensky series for the Molodensky problem, on the one hand, and for the GPS boundary problem, on the other hand, is very clear because Δg and δg have the same analytical and geometric structure.

At the same time, this similarity is very surprising since the two underlying boundary problems are mathematically quite different, as we have seen in sec. 2, compare eqs, (4) and (5), Nonetheless, (74) does give the potential as (5) requires: by Bruns' theorem, which is the omnipresent link between geometry and physics, we have

$$T = \gamma\zeta \quad . \tag{79}$$

Then

$$W = U + T \tag{80}$$

Is the geopotential required by (5), and

$$C = W_0 - W \qquad (81)$$

Is the *geopotential number,* the physical measure of height above sea level, conventionally obtained by the cumbersome method of leveling, but now computed in a direct way from gravity data. (This is, of course, the physical, more general, equivalent of the geometric determination of the normal height by $H^* = h - \zeta$, according to eq. (9).)

All this shows the powerful and very generally applicable ideas of M.S. Molodensky.

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